





**GALAHAD** FDC

USER DOCUMENTATION

GALAHAD Optimization Library version 4.0

#### 1 SUMMARY

Given an under-determined set of linear equations/constraints  $\mathbf{a}_i^T \mathbf{x} = b_i$ , i = 1, ..., m involving  $n \ge m$  unknowns  $\mathbf{x}$ , this package determines whether the constraints are consistent, and if so how many of the constraints are dependent; a list of dependent constraints, that is, those which may be removed without changing the solution set, will be found and the remaining  $\mathbf{a}_i$  will be linearly independent. Full advantage is taken of any zero coefficients in the vectors  $\mathbf{a}_i$ .

ATTRIBUTES — Versions: GALAHAD\_FDC\_single, GALAHAD\_FDC\_double. Uses: GALAHAD\_CLOCK, GALAHAD\_SYMBOLS, GALAHAD\_STRING, GALAHAD\_SMT, GALAHAD\_ROOTS, GALAHAD\_SLS, GALAHAD\_ULS, GALAHAD\_SPECFILE, GALAHAD\_SPACE. Date: August 2006. Origin: N. I. M. Gould, Rutherford Appleton Laboratory. Language: Fortran 95 + TR 15581 or Fortran 2003. Parallelism: Some options may use OpenMP and its runtime library.

# 2 HOW TO USE THE PACKAGE

Access to the package requires a USE statement such as

Single precision version

USE GALAHAD\_FDC\_single

Double precision version

USE GALAHAD\_FDC\_double

If it is required to use both modules at the same time, the derived types SMT\_type, FDC\_time\_type, FDC\_control\_type, and FDC\_inform\_type (Section 2.3) and the subroutines FDC\_initialize, FDC\_find\_dependent, FDC\_terminate (Section 2.4) and FDC\_read\_specfile (Section 2.6) must be renamed on one of the USE statements.

### 2.1 Integer kinds

We use the term long INTEGER to denote INTEGER (kind=long), where long = selected\_int\_kind(18)).

## 2.2 OpenMP

OpenMP may be used by the GALAHAD\_FDC package to provide parallelism for some solver options in shared memory environments. See the documentation for the GALAHAD package SLS for more details. To run in parallel, OpenMP must be enabled at compilation time by using the correct compiler flag (usually some variant of -openmp). The number of threads may be controlled at runtime by setting the environment variable OMP\_NUM\_THREADS.

The code may be compiled and run in serial mode.

# 2.3 The derived data types

Three derived data types are accessible from the package.

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### 2.3.1 The derived data type for holding control parameters

The derived data type FDC\_control\_type is used to hold controlling data. Default values may be obtained by calling FDC\_initialize (see Section 2.4.1), while components may also be changed by calling GALAHAD\_FDC\_read\_spec (see Section 2.6.1). The components of FDC\_control\_type are:

- error is a scalar variable of type default INTEGER, that holds the stream number for error messages. Printing of error messages in FDC\_find\_dependent and FDC\_terminate is suppressed if error  $\leq 0$ . The default is error = 6.
- out is a scalar variable of type default INTEGER, that holds the stream number for informational messages. Printing of informational messages in FDC\_find\_dependent is suppressed if out < 0. The default is out = 6.
- print\_level is a scalar variable of type default INTEGER, that is used to control the amount of informational output which is required. No informational output will occur if print\_level  $\leq 0$ . If print\_level = 1, basic statistics of the performance of the package will be produced. If print\_level  $\geq 2$ , this output will be increased to provide details such as the size of each neglected pivot. The default is print\_level = 0.
- max\_infeas is a scalar variable of type default REAL (double precision in GALAHAD\_FDC\_double), that holds the largest permitted infeasibility for a dependent constraint. Specifically, if  $\mathbf{x}$  satisfies  $\mathbf{a}_i^T\mathbf{x} = c_i$  for the constraints deemed to be linearly independent, it is required that  $|\mathbf{a}_i^T\mathbf{x} c_i| \leq \max_{i=1}^{T} \mathbf{a}_i$  for those classified as dependent. The default is  $\max_{i=1}^{T} \mathbf{a}_i = u^{1/3}$ , where u is EPSILON (1.00) (EPSILON (1.000) in GALAHAD\_FDC\_double).
- pivot\_tol is a scalar variable of type default REAL (double precision in GALAHAD\_FDC\_double), that holds the relative pivot tolerance used by the matrix factorization when attempting to detect linearly dependent constraints. See the documentation for the packages SLS and ULS for details. The default is pivot\_tol = 0.5.
- use\_sls is a scalar variable of type default LOGICAL, that should be set .TRUE. if the GALAHAD package SLS is to be used to detect linearly dependent constraints, or .FALSE. if the GALAHAD package ULS is to be used instead. The default is use\_sls = .FALSE..
- scale is a scalar variable of type default LOGICAL, that must be set .TRUE. if the rows of **A** are to be scaled to have unit (infinity) norm and .FALSE. otherwise. The default is scale = .FALSE..
- space\_critical is a scalar variable of type default LOGICAL, that must be set .TRUE. if space is critical when allocating arrays and .FALSE. otherwise. The package may run faster if space\_critical is .FALSE. but at the possible expense of a larger storage requirement. The default is space\_critical = .FALSE..
- deallocate\_error\_fatal is a scalar variable of type default LOGICAL, that must be set .TRUE. if the user wishes to terminate execution if a deallocation fails, and .FALSE. if an attempt to continue will be made. The default is deallocate\_error\_fatal = .FALSE..
- symmetric\_linear\_solver is a scalar variable of type default CHARACTER and length 30, that specifies the external package to be used to solve any symmetric linear system that might arise. Current possible choices are 'sils', 'ma27', 'ma57', 'ma77', 'ma86', 'ma97', ssids, 'pardiso' and 'wsmp', although only 'sils' and, for OMP 4.0-compliant compilers, 'ssids' are installed by default. See the documentation for the GALAHAD package SLS for further details. The default is symmetric\_linear\_solver = 'sils', but we recommend 'ma97' if it available.
- unsymmetric\_linear\_solver is a scalar variable of type default CHARACTER and length 30, that specifies the external package to be used to solve any unsymmetric linear system that might arise. Current possible choices are 'gls', 'ma28' and 'ma48'. See the documentation for the GALAHAD package ULS for further details. The default is unsymmetric\_linear\_solver = 'gls', but we recommend 'ma48' if it available.

- prefix is a scalar variable of type default CHARACTER and length 30, that may be used to provide a user-selected character string to preface every line of printed output. Specifically, each line of output will be prefaced by the string prefix(2:LEN(TRIM(prefix))-1), thus ignoreing the first and last non-null components of the supplied string. If the user does not want to preface lines by such a string, they may use the default prefix = "".
- SLS\_control is a scalar variable argument of type SLS\_control\_type that is used to pass control options to external packages used to factorize relevant symmetric matrices that arise. See the documentation for the GALAHAD package SLS for further details. In particular, default values are as for SLS, except that SLS\_control%relative\_pivot\_tolerance is reset to pivot\_tol.
- ULS\_control is a scalar variable argument of type ULS\_control\_type that is used to pass control options to external packages used to factorize relevant unsymmetric matrices that arise. See the documentation for the GALAHAD package ULS for further details. In particular, default values are as for ULS, except that ULS\_control%relative\_pivot\_tolerance is reset to pivot\_tol.

# 2.3.2 The derived data type for holding timing information

The derived data type FDC\_time\_type is used to hold elapsed CPU and system clock times for the various parts of the calculation. The components of FDC\_time\_type are:

- total is a scalar variable of type default REAL (double precision in GALAHAD\_FDC\_double), that gives the total CPU time spent in the package.
- analyse is a scalar variable of type default REAL (double precision in GALAHAD\_FDC\_double), that gives the CPU time spent analysing the required matrices prior to factorization.
- factorize is a scalar variable of type default REAL (double precision in GALAHAD\_FDC\_double), that gives the CPU time spent factorizing the required matrices.
- clock\_total is a scalar variable of type default REAL (double precision in GALAHAD\_FDC\_double), that gives the total elapsed system clock time spent in the package.
- clock\_analyse is a scalar variable of type default REAL (double precision in GALAHAD\_FDC\_double), that gives the elapsed system clock time spent analysing the required matrices prior to factorization.
- clock\_factorize is a scalar variable of type default REAL (double precision in GALAHAD\_FDC\_double), that gives the elapsed system clock time spent factorizing the required matrices.

#### 2.3.3 The derived data type for holding informational parameters

The derived data type FDC\_inform\_type is used to hold parameters that give information about the progress and needs of the algorithm. The components of FDC\_inform\_type are:

- status is a scalar variable of type default INTEGER, that gives the exit status of the algorithm. See Section 2.5 for details.
- alloc\_status is a scalar variable of type default INTEGER, that gives the status of the last attempted array allocation or deallocation. This will be 0 if status = 0.
- bad\_alloc is a scalar variable of type default CHARACTER and length 80, that gives the name of the last internal array for which there were allocation or deallocation errors. This will be the null string if status = 0.
- factorization\_status is a scalar variable of type default INTEGER, that gives the return status from the matrix factorization.

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factorization\_integer is a scalar variable of type long INTEGER, that gives the amount of integer storage used for the matrix factorization.

- factorization\_real is a scalar variable of type long INTEGER, that gives the amount of real storage used for the matrix factorization.
- non\_negligible\_pivot is a scalar variable of type default REAL (double precision in GALAHAD\_FDC\_double), that holds the value of the smallest pivot larger than control%zero\_pivot when searching for dependent linear constraints. If non\_negligible\_pivot is close to control%zero\_pivot, this may indicate that there are further dependent constraints, and it may be worth increasing control%zero\_pivot above non\_negligible\_pivot and solving again.
- time is a scalar variable of type FDC\_time\_type whose components are used to hold elapsed CPU and system clock times for the various parts of the calculation (see Section 2.3.2).
- SLS\_inform is a scalar variable argument of type SLS\_inform\_type that is used to pass information concerning the progress of the external packages used to factorize relevant symmetric matrices that arise. See the documentation for the GALAHAD package SLS for further details.
- ULS\_inform is a scalar variable argument of type ULS\_inform\_type that is used to pass information concerning the progress of the external packages used to factorize relevant symmetric matrices that arise. See the documentation for the GALAHAD package ULS for further details.

## 2.4 Argument lists and calling sequences

There are three procedures for user calls (see Section 2.6 for further features):

- 1. The subroutine FDC\_initialize is used to set default values before attempting to identify dependent constraints.
- 2. The subroutine FDC\_find\_dependent is called to identify dependent constraints.
- 3. The subroutine FDC\_terminate is provided to allow the user to automatically deallocate workspace array components, previously allocated by FDC\_find\_dependent, after use.

We use square brackets [ ] to indicate OPTIONAL arguments.

# 2.4.1 The initialization subroutine

Default values are provided as follows:

```
CALL FDC_initialize( data, control, inform )
```

- data is a scalar INTENT (INOUT) argument of type FDC\_data\_type that need not be set on entry and whose components will be used as workspace.
- control is a scalar INTENT (OUT) argument of type FDC\_control\_type (see Section 2.3.1). On exit, control contains default values for the components as described in Section 2.3.1. These values should only be changed after calling FDC\_initialize.
- inform is a scalar INTENT (INOUT) argument of type FDC\_inform\_type (see Section 2.3.3). A successful call to FDC\_find\_dependent is indicated when the component status has the value 0. For other return values of status, see Section 2.5.

#### 2.4.2 The subroutine for finding dependent constraints

Dependent constraints are identified as follows:

```
CALL FDC_find_dependent( n, m, A_val, A_col, A_ptr, B, & n_depen, DEPEN, data, control, inform )
```

- m is a scalar INTENT (IN) argument of type default INTEGER, that holds the number of constraints, m. **Restriction:** m > 0.
- n is a scalar INTENT (IN) argument of type default INTEGER, that holds the number of unknowns, n. **Restriction:**  $n \ge 0$ .
- A\_val is an INTENT(IN) rank-one array of type default REAL (double precision in GALAHAD\_FDC\_double), that holds the values of the entries (that is those component whose values are nonzero) of the matrix  $\mathbf{A}$  whose rows are the vectors  $\mathbf{a}_i^T$ ,  $i=1,\ldots,m$ . The entries for row i must directly precede those in row i, but the order within each row is unimportant.
- A\_col is an INTENT (IN) rank-one array of type default INTEGER, that holds the (column) indices of the entries of A corresponding to the values input in A\_val.
- A\_ptr is an INTENT(IN) rank-one array of dimension m+1 and type default INTEGER, whose *i*-th entry holds the starting position of row *i* of **A** for i = 1, ..., m. The m + 1-st entry of A\_ptr must hold the total number of entries plus one.
- is an INTENT (IN) rank-one array of dimension m and type default REAL (double precision in GALAHAD\_FDC\_double), whose *i*-th component must be set to  $b_i$  for i = 1, ..., m.
- n\_depen is a scalar INTENT (OUT) argument of type default INTEGER, that gives the number of dependent constraints.
- DEPEN is a rank-one allocatable array of type default INTEGER, On exit, if n\_depen > 0, it will have been allocated to be of length n\_depen and its components will be the indices of the dependent constraints. It will not be allocated or set if n\_depen = 0.
- data is a scalar INTENT (INOUT) argument of type FDC\_data\_type that need not be set on entry and whose components will be used as workspace.
- control is a scalar INTENT(IN) argument of type FDC\_control\_type (see Section 2.3.1). Default values may be assigned by calling FDC\_initialize prior to the first call to FDC\_find\_dependent.
- inform is a scalar INTENT (INOUT) argument of type FDC\_inform\_type (see Section 2.3.3). A successful call to FDC\_find\_dependent is indicated when the component status has the value 0. For other return values of status, see Section 2.5.

#### 2.4.3 The termination subroutine

All previously allocated arrays are deallocated as follows:

```
CALL FDC_terminate( data, control, inform[, C_depen ])
```

- data is a scalar INTENT (INOUT) argument of type FDC\_data\_type whose array components will be deallocated on exit.
- control is a scalar INTENT(IN) argument of type FDC\_control\_type exactly as for FDC\_find\_dependent.
- inform is a scalar INTENT (INOUT) argument of type FDC\_inform\_type exactly as for FDC\_find\_dependent. Only the components status alloc\_status and bad\_alloc will be set on exit, and a successful call to FDC\_terminate is indicated when this component status has the value 0. For other return values of status, see Section 2.5.

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C\_depen is an OPTIONAL rank-one allocatable array of type default INTEGER, that will be deallocated on exit if PRESENT.

#### 2.5 Warning and error messages

A negative value of inform%status on exit from FDC\_find\_dependent or FDC\_terminate indicates that an error has occurred. No further calls should be made until the error has been corrected. Possible values are:

- -1. An allocation error occurred. A message indicating the offending array is written on unit control%error, and the returned allocation status and a string containing the name of the offending array are held in inform%alloc\_status and inform%bad\_alloc respectively.
- -2. A deallocation error occurred. A message indicating the offending array is written on unit control%error and the returned allocation status and a string containing the name of the offending array are held in inform%alloc\_-status and inform%bad\_alloc respectively. status is given by the value inform%alloc\_status.
- -3. One of the restrictions n > 0 or m > 0 has been violated.
- -5 The constraints appear to be inconsistent.
- -9. The analysis phase of the factorization failed; the return status from the factorization package is given in the component inform%factorization\_status.
- -10. The factorization failed; the return status from the factorization package is given in the component inform%factorization\_status.

### 2.6 Further features

In this section, we describe an alternative means of setting control parameters, that is components of the variable control of type FDC\_control\_type (see Section 2.3.1), by reading an appropriate data specification file using the subroutine FDC\_read\_specfile. This facility is useful as it allows a user to change FDC control parameters without editing and recompiling programs that call FDC.

A specification file, or specfile, is a data file containing a number of "specification commands". Each command occurs on a separate line, and comprises a "keyword", which is a string (in a close-to-natural language) used to identify a control parameter, and an (optional) "value", which defines the value to be assigned to the given control parameter. All keywords and values are case insensitive, keywords may be preceded by one or more blanks but values must not contain blanks, and each value must be separated from its keyword by at least one blank. Values must not contain more than 30 characters, and each line of the specfile is limited to 80 characters, including the blanks separating keyword and value.

The portion of the specification file used by FDC\_read\_specfile must start with a "BEGIN FDC" command and end with an "END" command. The syntax of the specifile is thus defined as follows:

```
( .. lines ignored by FDC_read_specfile .. )
BEGIN FDC
    keyword value
    .....
    keyword value
END
( .. lines ignored by FDC_read_specfile .. )
```

where keyword and value are two strings separated by (at least) one blank. The "BEGIN FDC" and "END" delimiter command lines may contain additional (trailing) strings so long as such strings are separated by one or more blanks, so that lines such as

BEGIN FDC SPECIFICATION

and

END FDC SPECIFICATION

are acceptable. Furthermore, between the "BEGIN FDC" and "END" delimiters, specification commands may occur in any order. Blank lines and lines whose first non-blank character is ! or \* are ignored. The content of a line after a ! or \* character is also ignored (as is the ! or \* character itself). This provides an easy manner to "comment out" some specification commands, or to comment specific values of certain control parameters.

The value of a control parameters may be of three different types, namely integer, logical or real. Integer and real values may be expressed in any relevant Fortran integer and floating-point formats (respectively). Permitted values for logical parameters are "ON", "TRUE", ".TRUE.", "T", "YES", "Y", or "OFF", "NO", "N", "FALSE", ".FALSE." and "F". Empty values are also allowed for logical control parameters, and are interpreted as "TRUE".

The specification file must be open for input when FDC\_read\_specfile is called, and the associated device number passed to the routine in device (see below). Note that the corresponding file is REWINDed, which makes it possible to combine the specifications for more than one program/routine. For the same reason, the file is not closed by FDC\_read\_specfile.

### 2.6.1 To read control parameters from a specification file

Control parameters may be read from a file as follows:

```
CALL FDC_read_specfile( control, device )
```

control is a scalar INTENT (INOUT) argument of type FDC\_control\_type (see Section 2.3.1). Default values should have already been set, perhaps by calling FDC\_initialize. On exit, individual components of control may have been changed according to the commands found in the specifile. Specifile commands and the component (see Section 2.3.1) of control that each affects are given in Table 2.1.

command	component of control	value type
error-printout-device	%error	integer
printout-device	%out	integer
print-level	%print_level	integer
maximum-permitted-infeasibility	%max_infeas	real
pivot-tolerance-used-for-dependencies	%pivot_tol	real
use-sls	%use_sls	logical
scale-A	%scale	logical
space-critical	%space_critical	logical
deallocate-error-fatal	%deallocate_error_fatal	logical
symmetric-linear-equation-solver	%symmetric_linear_solver	character
output-line-prefix	%prefix	character

Table 2.1: Specifle commands and associated components of control.

device is a scalar INTENT (IN) argument of type default INTEGER, that must be set to the unit number on which the specifile has been opened. If device is not open, control will not be altered and execution will continue, but an error message will be printed on unit control%error.

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#### 2.7 Information printed

If control%print\_level is positive, information about the progress of the algorithm will be printed on unit control%out. If control%print\_level = 1, basic statistics of the performance of the package will be produced. If control%print\_level  $\geq 2$  this output will be increased to provide details such as the size of each neglected pivot.

# 3 GENERAL INFORMATION

Use of common: None.

**Workspace:** Provided automatically by the module.

Other routines called directly: None.

Other modules used directly: FDC\_find\_dependent calls the GALAHAD packages GALAHAD\_CLOCK, GALAHAD\_SYMBOLS, GALAHAD\_STRING, GALAHAD\_SMT, GALAHAD\_ROOTS, GALAHAD\_SLS, GALAHAD\_ULS, GALAHAD\_SPECFILE and GALAHAD\_SPACE.

Input/output: Output is under control of the arguments control%error, control%out and control%print\_level.

**Restrictions:**  $n \ge 0$ ,  $m \ge 0$ .

**Portability:** ISO Fortran 95 + TR 15581 or Fortran 2003. The package is thread-safe.

### 4 METHOD

A choice of two methods is available. In the first, the matrix

$$\mathbf{K} = \left( \begin{array}{cc} \alpha \mathbf{I} & \mathbf{A}^T \\ \mathbf{A} & \mathbf{0} \end{array} \right)$$

is formed and factorized for some small  $\alpha > 0$  using the GALAHAD package SLS—the factors  $\mathbf{K} = \mathbf{PLDL}^T \mathbf{P}^T$  are used to determine whether  $\mathbf{A}$  has dependent rows. In particular, in exact arithmetic dependencies in  $\mathbf{A}$  will correspond to zero pivots in the block diagonal matrix  $\mathbf{D}$ .

The second choice of method finds factors  $\mathbf{A} = \mathbf{PLUQ}$  of the rectangular matrix  $\mathbf{A}$  using the GALAHAD package ULS. In this case, dependencies in  $\mathbf{A}$  will be reflected in zero diagonal entries in  $\mathbf{U}$  in exact arithmetic.

The factorization in either case may also be used to determine whether the system is consistent.

### 5 EXAMPLE OF USE

Suppose we wish to find whether the linear constraints  $x_1 + 2x_2 + 3x_3 + 4x_4 = 5$ ,  $2x_1 - 4x_2 + 6x_3 - 8x_4 = 10$  and  $5x_2 + 10x_4 = 0$  are consistent but redundant. Then we may use the following code.

```
INTEGER :: n_depen
TYPE ( FDC_data_type ) :: data
TYPE ( FDC_control_type ) :: control
TYPE ( FDC_inform_type ) :: inform
A_{val} = (/ 1.0_{wp}, 2.0_{wp}, 3.0_{wp}, 4.0_{wp}, 2.0_{wp}, -4.0_{wp}, 6.0_{wp},
          -8.0_wp, 5.0_wp, 10.0_wp /)
A_{col} = (/ 1, 2, 3, 4, 1, 2, 3, 4, 2, 4 /)
A_ptr = (/1, 5, 9, 11/)
B = (/ 5.0_{wp}, 10.0_{wp}, 0.0_{wp}/)
CALL FDC_initialize( data, control, inform ) ! Initialize control parameters
CALL FDC_find_dependent( n, m, A_val, A_col, A_ptr, B, n_depen, DEPEN,
                         data, control, inform ) ! Check for dependencies
IF ( inform%status == 0 ) THEN
                                             ! Successful return
  IF ( n_{depen} == 0 ) THEN
    WRITE( 6, "( ^{\prime} FDC_find_dependent - no dependencies ^{\prime} )")
    WRITE( 6, "( ' FDC_find_dependent - dependent constraint(s):', 3I3 )") &
     DEPEN
  END IF
ELSE! Error returns
  WRITE( 6, "( ' FDC_find_dependent exit status = ', I6 ) " ) inform%status
CALL FDC_terminate( data, control, inform, DEPEN ) ! Delete workspace
END PROGRAM GALAHAD_FDC_example
```

# This produces the following output:

```
FDC_find_dependent - dependent constraint(s): 1
```

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