



GALAHAD

QPC

USER DOCUMENTATION

GALAHAD Optimization Library version 4.0

1 SUMMARY

This package uses a crossover method, switching between interior-point and working-set approaches, to solve the **quadratic programming problem**

$$\text{minimize } q(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{g}^T\mathbf{x} + f$$

subject to the general linear constraints

$$c_i^l \leq \mathbf{a}_i^T \mathbf{x} \leq c_i^u, \quad i = 1, \dots, m,$$

and the simple bound constraints

$$x_j^l \leq x_j \leq x_j^u, \quad j = 1, \dots, n,$$

where the n by n symmetric matrix \mathbf{H} , the vectors \mathbf{g} , \mathbf{a}_i , \mathbf{c}^l , \mathbf{c}^u , \mathbf{x}^l , \mathbf{x}^u and the scalar f are given. Full advantage is taken of any zero coefficients in the matrix \mathbf{H} or the vectors \mathbf{a}_i . Any of the constraint bounds c_i^l , c_i^u , x_j^l and x_j^u may be infinite.

If the matrix \mathbf{H} is positive semi-definite, a global solution is found. However, if \mathbf{H} is indefinite, the procedure may find a (weak second-order) critical point that is not the global solution to the given problem.

ATTRIBUTES — Versions: GALAHAD_QPC_single, GALAHAD_QPC_double. **Uses:** GALAHAD_CLOCK, GALAHAD_SYMBOLS, GALAHAD_SPACE, GALAHAD_NORMS, GALAHAD_SORT, GALAHAD_QPT, GALAHAD_QPP, GALAHAD_QPD, GALAHAD_EQP, GALAHAD_LSQP, GALAHAD_CQP, GALAHAD_QPA, GALAHAD_QPB, GALAHAD_SPECFILE, GALAHAD_FDC. **Date:** March 2006. **Origin:** N. I. M. Gould, Rutherford Appleton Laboratory, and Ph. L. Toint, University of Namur, Belgium. **Language:** Fortran 95 + TR 15581 or Fortran 2003. **Parallelism:** Some options may use OpenMP and its runtime library.

2 HOW TO USE THE PACKAGE

Access to the package requires a USE statement such as

Single precision version

```
USE GALAHAD_QPC_single
```

Double precision version

```
USE GALAHAD_QPC_double
```

If it is required to use both modules at the same time, the derived types SMT_type, QPT_problem_type, QPC_time_type, QPC_control_type, QPC_inform_type and QPC_data_type (Section 2.4) and the subroutines QPC_initialize, QPC_solve, QPC_terminate, (Section 2.5) and QPC_read_specfile (Section 2.7) must be renamed on one of the USE statements.

2.1 Matrix storage formats

Both the Hessian matrix \mathbf{H} and the constraint Jacobian \mathbf{A} , the matrix whose rows are the vectors \mathbf{a}_i^T , $i = 1, \dots, m$, may be stored in a variety of input formats.

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2.1.1 Dense storage format

The matrix \mathbf{A} is stored as a compact dense matrix by rows, that is, the values of the entries of each row in turn are stored in order within an appropriate real one-dimensional array. Component $n * (i - 1) + j$ of the storage array `A%val` will hold the value a_{ij} for $i = 1, \dots, m$, $j = 1, \dots, n$. Since \mathbf{H} is symmetric, only the lower triangular part (that is the part h_{ij} for $1 \leq j \leq i \leq n$) need be held. In this case the lower triangle will be stored by rows, that is component $i * (i - 1) / 2 + j$ of the storage array `H%val` will hold the value h_{ij} (and, by symmetry, h_{ji}) for $1 \leq j \leq i \leq n$.

2.1.2 Sparse co-ordinate storage format

Only the nonzero entries of the matrices are stored. For the l -th entry of \mathbf{A} , its row index i , column index j and value a_{ij} are stored in the l -th components of the integer arrays `A%row`, `A%col` and real array `A%val`, respectively. The order is unimportant, but the total number of entries `A%ne` is also required. The same scheme is applicable to \mathbf{H} (thus requiring integer arrays `H%row`, `H%col`, a real array `H%val` and an integer value `H%ne`), except that only the entries in the lower triangle need be stored.

2.1.3 Sparse row-wise storage format

Again only the nonzero entries are stored, but this time they are ordered so that those in row i appear directly before those in row $i + 1$. For the i -th row of \mathbf{A} , the i -th component of an integer array `A%ptr` holds the position of the first entry in this row, while `A%ptr(m + 1)` holds the total number of entries plus one. The column indices j and values a_{ij} of the entries in the i -th row are stored in components $l = \text{A\%ptr}(i), \dots, \text{A\%ptr}(i + 1) - 1$ of the integer array `A%col`, and real array `A%val`, respectively. The same scheme is applicable to \mathbf{H} (thus requiring integer arrays `H%ptr`, `H%col`, and a real array `H%val`), except that only the entries in the lower triangle need be stored.

For sparse matrices, this scheme almost always requires less storage than its predecessor.

2.1.4 Diagonal storage format

If \mathbf{H} is diagonal (i.e., $h_{ij} = 0$ for all $1 \leq i \neq j \leq n$) only the diagonal entries h_{ii} , $1 \leq i \leq n$, need be stored, and the first n components of the array `H%val` may be used for the purpose. There is no sensible equivalent for the non-square \mathbf{A} .

2.2 Integer kinds

We use the term long INTEGER to denote `INTEGER(kind=long)`, where `long = selected_int_kind(18)`.

2.3 OpenMP

OpenMP may be used by the `GALAHAD_QPC` package to provide parallelism for some solver options in shared memory environments. See the documentation for the GALAHAD package SLS for more details. To run in parallel, OpenMP must be enabled at compilation time by using the correct compiler flag (usually some variant of `-openmp`). The number of threads may be controlled at runtime by setting the environment variable `OMP_NUM_THREADS`.

The code may be compiled and run in serial mode.

2.4 The derived data types

Ten derived data types are accessible from the package.

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2.4.1 The derived data type for holding matrices

The derived data type `SMT_TYPE` is used to hold the matrices **A** and **H**. The components of `SMT_TYPE` used here are:

- `m` is a scalar component of type default `INTEGER`, that holds the number of rows in the matrix.
- `n` is a scalar component of type default `INTEGER`, that holds the number of columns in the matrix.
- `ne` is a scalar variable of type default `INTEGER`, that holds the number of matrix entries.
- `type` is a rank-one allocatable array of type default `CHARACTER`, that is used to indicate the matrix storage scheme used. Its precise length and content depends on the type of matrix to be stored (see §2.4.2).
- `val` is a rank-one allocatable array of type default `REAL` (double precision in `GALAHAD_QPC_double`) and dimension at least `ne`, that holds the values of the entries. Each pair of off-diagonal entries $h_{ij} = h_{ji}$ of a *symmetric* matrix **H** is represented as a single entry (see §2.1.1–2.1.3). Any duplicated entries that appear in the sparse co-ordinate or row-wise schemes will be summed.
- `row` is a rank-one allocatable array of type default `INTEGER`, and dimension at least `ne`, that may hold the row indices of the entries. (see §2.1.2).
- `col` is a rank-one allocatable array of type default `INTEGER`, and dimension at least `ne`, that may the column indices of the entries (see §2.1.2–2.1.3).
- `ptr` is a rank-one allocatable array of type default `INTEGER`, and dimension at least `m + 1`, that may hold the pointers to the first entry in each row (see §2.1.3).

2.4.2 The derived data type for holding the problem

The derived data type `QPT_problem_type` is used to hold the problem. The components of `QPT_problem_type` are:

- `new_problem_structure` is a scalar variable of type default `LOGICAL`, that is `.TRUE.` if this is the first (or only) problem in a sequence of problems with identical "structure" to be attempted, and `.FALSE.` if a previous problem with the same "structure" (but different numerical data) has been solved. Here, the term "structure" refers both to the sparsity patterns of the Jacobian matrices **A** involved (but not their numerical values), to the zero/nonzero/infinity patterns (a bound is either zero, \pm infinity, or a finite but arbitrary nonzero) of each of the constraint bounds, and to the variables and constraints that are fixed (both bounds are the same) or free (the lower and upper bounds are \pm infinity, respectively).
- `n` is a scalar variable of type default `INTEGER`, that holds the number of optimization variables, n .
- `m` is a scalar variable of type default `INTEGER`, that holds the number of general linear constraints, m .
- `H` is scalar variable of type `SMT_TYPE` that holds the Hessian matrix **H**. The following components are used:

`H%type` is an allocatable array of rank one and type default `CHARACTER`, that is used to indicate the storage scheme used. If the dense storage scheme (see Section 2.1.1) is used, the first five components of `H%type` must contain the string `DENSE`. For the sparse co-ordinate scheme (see Section 2.1.2), the first ten components of `H%type` must contain the string `COORDINATE`, for the sparse row-wise storage scheme (see Section 2.1.3), the first fourteen components of `H%type` must contain the string `SPARSE_BY_ROWS`, and for the diagonal storage scheme (see Section 2.1.4), the first eight components of `H%type` must contain the string `DIAGONAL`.

For convenience, the procedure `SMT_put` may be used to allocate sufficient space and insert the required keyword into `H%type`. For example, if `prob` is of derived type `QPC_problem_type` and involves a Hessian we wish to store using the co-ordinate scheme, we may simply

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```
CALL SMT_put( prob%H%type, 'COORDINATE' )
```

See the documentation for the GALAHAD package SMT for further details on the use of SMT_put.

- H%ne is a scalar variable of type default INTEGER, that holds the number of entries in the **lower triangular** part of \mathbf{H} in the sparse co-ordinate storage scheme (see Section 2.1.2). It need not be set for any of the other three schemes.
- H%val is a rank-one allocatable array of type default REAL (double precision in GALAHAD_QPC_double), that holds the values of the entries of the **lower triangular** part of the Hessian matrix \mathbf{H} in any of the storage schemes discussed in Section 2.1.
- H%row is a rank-one allocatable array of type default INTEGER, that holds the row indices of the **lower triangular** part of \mathbf{H} in the sparse co-ordinate storage scheme (see Section 2.1.2). It need not be allocated for any of the other three schemes.
- H%col is a rank-one allocatable array variable of type default INTEGER, that holds the column indices of the **lower triangular** part of \mathbf{H} in either the sparse co-ordinate (see Section 2.1.2), or the sparse row-wise (see Section 2.1.3) storage scheme. It need not be allocated when the dense or diagonal storage schemes are used.
- H%ptr is a rank-one allocatable array of dimension $n+1$ and type default INTEGER, that holds the starting position of each row of the **lower triangular** part of \mathbf{H} , as well as the total number of entries plus one, in the sparse row-wise storage scheme (see Section 2.1.3). It need not be allocated when the other schemes are used.
- G is a rank-one allocatable array of dimension n and type default REAL (double precision in GALAHAD_QPC_double), that holds the gradient \mathbf{g} of the linear term of the quadratic objective function. The j -th component of G , $j = 1, \dots, n$, contains \mathbf{g}_j .
- f is a scalar variable of type default REAL (double precision in GALAHAD_QPC_double), that holds the constant term, f , in the objective function.
- A is scalar variable of type SMT_TYPE that holds the Jacobian matrix \mathbf{A} . The following components are used:
- A%type is an allocatable array of rank one and type default CHARACTER, that is used to indicate the storage scheme used. If the dense storage scheme (see Section 2.1.1) is used, the first five components of A%type must contain the string DENSE. For the sparse co-ordinate scheme (see Section 2.1.2), the first ten components of A%type must contain the string COORDINATE, while for the sparse row-wise storage scheme (see Section 2.1.3), the first fourteen components of A%type must contain the string SPARSE_BY_ROWS.
- Just as for H%type above, the procedure SMT_put may be used to allocate sufficient space and insert the required keyword into A%type. Once again, if prob is of derived type QPC_problem_type and involves a Jacobian we wish to store using the sparse row-wise storage scheme, we may simply
- ```
CALL SMT_put(prob%A%type, 'SPARSE_BY_ROWS')
```
- A%ne is a scalar variable of type default INTEGER, that holds the number of entries in  $\mathbf{A}$  in the sparse co-ordinate storage scheme (see Section 2.1.2). It need not be set for either of the other two schemes.
- A%val is a rank-one allocatable array of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the values of the entries of the Jacobian matrix  $\mathbf{A}$  in any of the storage schemes discussed in Section 2.1.
- A%row is a rank-one allocatable array of type default INTEGER, that holds the row indices of  $\mathbf{A}$  in the sparse co-ordinate storage scheme (see Section 2.1.2). It need not be allocated for either of the other two schemes.
- A%col is a rank-one allocatable array variable of type default INTEGER, that holds the column indices of  $\mathbf{A}$  in either the sparse co-ordinate (see Section 2.1.2), or the sparse row-wise (see Section 2.1.3) storage scheme. It need not be allocated when the dense storage scheme is used.

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- `A%ptr` is a rank-one allocatable array of dimension  $m+1$  and type default `INTEGER`, that holds the starting position of each row of  $\mathbf{A}$ , as well as the total number of entries plus one, in the sparse row-wise storage scheme (see Section 2.1.3). It need not be allocated when the other schemes are used.
- `C_l` is a rank-one allocatable array of dimension  $m$  and type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the vector of lower bounds  $\mathbf{c}^l$  on the general constraints. The  $i$ -th component of `C_l`,  $i = 1, \dots, m$ , contains  $c_i^l$ . Infinite bounds are allowed by setting the corresponding components of `C_l` to any value smaller than `-infinity`, where `infinity` is a component of the control array `control` (see Section 2.4.7).
- `C_u` is a rank-one allocatable array of dimension  $m$  and type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the vector of upper bounds  $\mathbf{c}^u$  on the general constraints. The  $i$ -th component of `C_u`,  $i = 1, \dots, m$ , contains  $c_i^u$ . Infinite bounds are allowed by setting the corresponding components of `C_u` to any value larger than `infinity`, where `infinity` is a component of the control array `control` (see Section 2.4.7).
- `X_l` is a rank-one allocatable array of dimension  $n$  and type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the vector of lower bounds  $\mathbf{x}^l$  on the variables. The  $j$ -th component of `X_l`,  $j = 1, \dots, n$ , contains  $x_j^l$ . Infinite bounds are allowed by setting the corresponding components of `X_l` to any value smaller than `-infinity`, where `infinity` is a component of the control array `control` (see Section 2.4.7).
- `X_u` is a rank-one allocatable array of dimension  $n$  and type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the vector of upper bounds  $\mathbf{x}^u$  on the variables. The  $j$ -th component of `X_u`,  $j = 1, \dots, n$ , contains  $x_j^u$ . Infinite bounds are allowed by setting the corresponding components of `X_u` to any value larger than that `infinity`, where `infinity` is a component of the control array `control` (see Section 2.4.7).
- `X` is a rank-one allocatable array of dimension  $n$  and type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the values  $\mathbf{x}$  of the optimization variables. The  $j$ -th component of `X`,  $j = 1, \dots, n$ , contains  $x_j$ .
- `Z` is a rank-one allocatable array of dimension  $n$  and type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the values  $\mathbf{z}$  of estimates of the dual variables corresponding to the simple bound constraints (see Section 4). The  $j$ -th component of `Z`,  $j = 1, \dots, n$ , contains  $z_j$ .
- `C` is a rank-one allocatable array of dimension  $m$  and type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the values  $\mathbf{Ax}$  of the constraints. The  $i$ -th component of `C`,  $i = 1, \dots, m$ , contains  $\mathbf{a}_i^T \mathbf{x} \equiv (\mathbf{Ax})_i$ .
- `Y` is a rank-one allocatable array of dimension  $m$  and type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the values  $\mathbf{y}$  of estimates of the Lagrange multipliers corresponding to the general linear constraints (see Section 4). The  $i$ -th component of `Y`,  $i = 1, \dots, m$ , contains  $y_i$ .

### 2.4.3 The derived data type for holding control parameters specifically for the interior-point phase

The derived data type `QPB_control_type` is used to hold controlling data for the interior-point phase of the calculation, performed by the GALAHAD package `GALAHAD_QPB`. Components may be changed by calling `QPB_read_specfile` (see the documentation for `GALAHAD_QPB` for details). The components of `QPB_control_type` are:

- `error` is a scalar variable of type default `INTEGER`, that holds the stream number for error messages. Printing of error messages in `QPB_solve` and `QPB_terminate` is suppressed if `error`  $\leq 0$ . The default is `error` = 6.
- `out` is a scalar variable of type default `INTEGER`, that holds the stream number for informational messages. Printing of informational messages in `QPB_solve` is suppressed if `out`  $< 0$ . The default is `out` = 6.
- `print_level` is a scalar variable of type default `INTEGER`, that is used to control the amount of informational output which is required. No informational output will occur if `print_level`  $\leq 0$ . If `print_level` = 1, a single line of output will be produced for each iteration of the process. If `print_level`  $\geq 2$ , this output will be increased to provide significant detail of each iteration. The default is `print_level` = 0.

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`start_print` is a scalar variable of type default `INTEGER`, that specifies the first iteration for which printing will occur in `QPB_solve`. If `start_print` is negative, printing will occur from the outset. The default is `start_print = -1`.

`stop_print` is a scalar variable of type default `INTEGER`, that specifies the last iteration for which printing will occur in `QPB_solve`. If `stop_print` is negative, printing will occur once it has been started by `start_print`. The default is `stop_print = -1`.

`maxit` is a scalar variable of type default `INTEGER`, that holds the maximum number of iterations which will be allowed in `QPB_solve`. The default is `maxit = 1000`.

`infeas_max` is a scalar variable of type default `INTEGER`, that specifies the number of iterations for which the overall infeasibility of the problem is not reduced by at least a factor `reduce_infeas` before the problem is flagged as infeasible (see `reduce_infeas`). The default is `infeas_max = 200`.

`cg_maxit` is a scalar variable of type default `INTEGER`, that holds the maximum number of conjugate-gradient inner iterations that may be performed during the computation of each search direction in `QPB_solve`. If `cg_maxit` is set to a negative number, it will be reset by `QPB_solve` to the dimension of the relevant linear system +1. The default is `cg_maxit = 200`.

`indicator_type` is a scalar variable of type default `INTEGER`, that specifies the type of indicator used to assess when a variable or constraint is active. Possible values are:

- 1 a variable/constraint is active if and only if the distance to its nearest bound is no larger than `indicator_tol_p` (see below).
- 2 a variable/constraint is active if and only if the distance to its nearest bound is no larger than `indicator_tol_pd` (see below) times the magnitude of its corresponding dual variable.
- 3 a variable/constraint is active if and only if the distance to its nearest bound is no larger than `indicator_tol_tapia` (see below) times the distance to the same bound at the previous iteration.

The default is `indicator_type = 3`.

`restore_problem` is a scalar variable of type default `INTEGER`, that specifies how much of the input problem is to be restored on output. Possible values are:

- 0 nothing is restored.
- 1 the vector data  $\mathbf{g}$ ,  $\mathbf{c}^l$ ,  $\mathbf{c}^u$ ,  $\mathbf{x}^l$ , and  $\mathbf{x}^u$  will be restored to their input values.
- 2 the entire problem, that is the above vector data along with the Hessian matrix  $\mathbf{H}$  and the Jacobian matrix  $\mathbf{A}$ , will be restored.

The default is `restore_problem = 2`.

`extrapolate` is a scalar variable of type default `INTEGER`, that specifies whether extrapolation should be used to track the central path. Possible values are:

- 0 no extrapolation should be used.
- 1 extrapolation should be used after the final update of the barrier parameter.
- 2 extrapolation should be used after each update of the barrier parameter.

The default is `extrapolate = 2`.

`path_history` is a scalar variable of type default `INTEGER`, that specifies the maximum number of previous path points used when extrapolating to track the central path. The default is `path_history = 1`.

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`path_derivatives` is a scalar variable of type default INTEGER, that specifies the maximum order of derivatives to use when extrapolating to track the central path. The default is `path_derivatives = 5`.

`fit_order` is a scalar variable of type default INTEGER, that specifies the order of (Puiseux or Taylor) series to fit to the available data when building the extrapolant to track the central path. If `fit_order ≤ 0`, the order will be such that all available data as specified by `path_history` and `path_derivatives` will be used. The default is `fit_order = - 1`.

`infinity` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that is used to specify which constraint bounds are infinite. Any bound larger than `infinity` in modulus will be regarded as infinite. The default is `infinity = 1019`.

`stop_p` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the required accuracy for the primal infeasibility (see Section 4). The default is `stop_p =  $u^{1/3}$` , where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in GALAHAD\_QPC\_double).

`stop_d` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the required accuracy for the dual infeasibility (see Section 4). The default is `stop_d =  $u^{1/3}$` , where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in GALAHAD\_QPC\_double).

`stop_c` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the required accuracy for the violation of complementarity slackness (see Section 4). The default is `stop_c =  $u^{1/3}$` , where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in GALAHAD\_QPC\_double).

`prfeas` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that aims to specify the closest that any initial variable may be to infeasibility. Any variable closer to infeasibility than `prfeas` will be moved to `prfeas` from the offending bound. However, if a variable is range bounded, and its bounds are closer than `prfeas` apart, it will be moved to the mid-point of the two bounds. The default is `prfeas = 1.0`.

`dufeas` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that aims to specify the closest that any initial dual variable or Lagrange multiplier may be to infeasibility. Any variable closer to infeasibility than `prfeas` will be moved to `dufeas` from the offending bound. However, if a dual variable is range bounded, and its bounds are closer than `dufeas` apart, it will be moved to the mid-point of the two bounds. The default is `dufeas = 1.0`.

`muzero` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the initial value of the barrier parameter. If `muzero` is not positive, it will be reset automatically to an appropriate value. The default is `muzero = -1.0`.

`reduce_infeas` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies the least factor by which the overall infeasibility of the problem must be reduced, over `infeas_max` consecutive iterations, for it not be declared infeasible (see `infeas_max`). The default is `reduce_infeas = 0.99`.

`obj_unbounded` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies smallest value of the objective function that will be tolerated before the problem is declared to be unbounded from below. The default is `obj_unbounded =  $-u^{-2}$` , where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in GALAHAD\_QPC\_double).

`identical_bounds_tol` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double). Every pair of constraint bounds  $(c_i^l, c_i^u)$  or  $(x_j^l, x_j^u)$  that is closer than `identical_bounds_tol` will be reset to the average of their values,  $\frac{1}{2}(c_i^l + c_i^u)$  or  $\frac{1}{2}(x_j^l + x_j^u)$  respectively. The default is `identical_bounds_tol =  $u$` , where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in GALAHAD\_QPC\_double).

`indicator_tol_p` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double) that provides the indicator tolerance associated with the test `indicator_type = 1`. The default is `indicator_tol_p =  $u^{1/3}$` , where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in GALAHAD\_QPC\_double).

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`indicator_tol_pd` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double) that provides the indicator tolerance associated with the test `indicator_type = 2`. The default is `indicator_tol_pd = 1.0`.

`indicator_tol_tapia` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double) that provides the indicator tolerance associated with the test `indicator_type = 3`. The default is `indicator_tol_tapia = 0.9`.

`inner_stop_relative` and `inner_stop_absolute` are scalar variables of type default REAL (double precision in GALAHAD\_QPC\_double), that hold the relative and absolute convergence tolerances for the inner iteration (search direction) problem using the package GALAHAD\_GLTR, and correspond to the values `control%stop_relative` and `control%stop_absolute` in that package. The defaults are `inner_stop_relative = 0.01` and `inner_stop_absolute =  $\sqrt{u}$` , where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in GALAHAD\_QPC\_double).

`initial_radius` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies the initial trust-region radius. If `initial_radius` is not positive, it will be reset automatically to an appropriate value. The default is `initial_radius = - 1.0`.

`inner_fraction_opt` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies the fraction of the optimal value which is acceptable for the solution of the inner iteration (search direction) problem using the package GALAHAD\_GLTR, and corresponds to the value `control%fraction_opt` in that package. A negative value is considered to be zero, and a value of larger than one is considered to be one. Reducing `fraction_opt` below one will result in a reduction of the computation performed at the expense of an inferior optimal value. The default is `inner_fraction_opt = 0.1`.

`cpu_time_limit` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that is used to specify the maximum permitted CPU time. Any negative value indicates no limit will be imposed. The default is `cpu_time_limit = - 1.0`.

`clock_time_limit` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that is used to specify the maximum permitted elapsed system clock time. Any negative value indicates no limit will be imposed. The default is `clock_time_limit = - 1.0`.

`remove_dependencies` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the algorithm is to attempt to remove any linearly dependent constraints before solving the problem, and `.FALSE.` otherwise. We recommend removing linearly dependencies. The default is `remove_dependencies = .TRUE..`

`treat_zero_bounds_as_general` is a scalar variable of type default LOGICAL. If it is set to `.FALSE.`, variables which are only bounded on one side, and whose bound is zero, will be recognised as non-negativities/non-positivities rather than simply as lower- or upper-bounded variables. If it is set to `.TRUE.`, any variable bound  $x_j^l$  or  $x_j^u$  which has the value 0.0 will be treated as if it had a general value. Setting `treat_zero_bounds_as_general` to `.TRUE.` has the advantage that if a sequence of problems are reordered, then bounds which are “accidentally” zero will be considered to have the same structure as those which are nonzero. However, GALAHAD\_QPC is able to take special advantage of non-negativities/non-positivities, so if a single problem, or if a sequence of problems whose bound structure is known not to change, is/are to be solved, it will pay to set the variable to `.FALSE..` The default is `treat_zero_bounds_as_general = .FALSE..`

`center` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the algorithm will should use the analytic center of the feasible set as its initial feasible point, and `.FALSE.` otherwise. We recommend using the analytic center. The default is `center = .TRUE..`

`primal` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if a primal-barrier Hessian will be used and `.FALSE.` if the primal-dual Hessian is preferred. We recommend using the primal-dual Hessian. The default is `primal = .FALSE..`

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`puiseux` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if a Puiseux series will be used when extrapolating along the central path and `.FALSE.` if a Taylor series is preferred. We recommend using the Puiseux series unless the solution is known to be non-degenerate. The default is `puiseux = .TRUE.`.

`feasol` is a scalar variable of type default LOGICAL, that should be set `.TRUE.` if the final solution obtained will be perturbed so that variables close to their bounds are moved onto these bounds, and `.FALSE.` otherwise. The default is `feasol = .FALSE.`.

`prefix` is a scalar variable of type default CHARACTER and length 30, that may be used to provide a user-selected character string to preface every line of printed output. Specifically, each line of output will be prefaced by the string `prefix(2:LEN(TRIM(prefix))-1)`, thus ignoring the first and last non-null components of the supplied string. If the user does not want to preface lines by such a string, they may use the default `prefix = ""`.

`LSQP_control` is a scalar variable of type `LSQP_control_type` whose components are used to control the initial feasible point calculation performed by the package `GALAHAD_LSQP`. See the specification sheet for the package `GALAHAD_LSQP` for details, and appropriate default values (but note that default value for `LSQP_control%feasol` is changed to `.FALSE.`).

`FDC_control` is a scalar variable of type `FDC_control_type` whose components are used to control any detection of linear dependencies performed by the package `GALAHAD_FDC`. See the specification sheet for the package `GALAHAD_FDC` for details, and appropriate default values.

`SBLS_control` is a scalar variable of type `SBLS_control_type` whose components are used to control preconditioning, needed to accelerate the step calculation, performed by the package `GALAHAD_SBLS`. See the specification sheet for the package `GALAHAD_SBLS` for details, and appropriate default values (but note that if `SBLS_control%preconditioner = 5`, the diagonal matrix used is corresponds to replacing the Hessian matrix by its barrier terms).

`GLTR_control` is a scalar variable of type `GLTR_control_type` whose components are used to control the step calculation performed by the package `GALAHAD_GLTR`. See the specification sheet for the package `GALAHAD_GLTR` for details, and appropriate default values.

#### 2.4.4 The derived data type for holding control parameters specifically for the interior-point phase in the convex case

The derived data type `CQP_control_type` is used to hold controlling data for the interior-point phase of the calculation for convex problems, performed by the `GALAHAD` package `GALAHAD_CQP`. Components may be changed by calling `CQP_read_specfile` (see the documentation for `GALAHAD_CQP` for details). The components of `CQP_control_type` are:

`error` is a scalar variable of type default INTEGER, that holds the stream number for error messages. Printing of error messages in `QPC_solve` and `QPC_terminate` is suppressed if `error ≤ 0`. The default is `error = 6`.

`out` is a scalar variable of type default INTEGER, that holds the stream number for informational messages. Printing of informational messages in `QPC_solve` is suppressed if `out < 0`. The default is `out = 6`.

`print_level` is a scalar variable of type default INTEGER, that is used to control the amount of informational output which is required. No informational output will occur if `print_level ≤ 0`. If `print_level = 1`, a single line of output will be produced for each iteration of the process. If `print_level ≥ 2`, this output will be increased to provide significant detail of each iteration. The default is `print_level = 0`.

`maxit` is a scalar variable of type default INTEGER, that holds the maximum number of iterations which will be allowed in `QPC_solve`. The default is `maxit = 1000`.

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`start_print` is a scalar variable of type default `INTEGER`, that specifies the first iteration for which printing will occur in `QPC_solve`. If `start_print` is negative, printing will occur from the outset. The default is `start_print = -1`.

`stop_print` is a scalar variable of type default `INTEGER`, that specifies the last iteration for which printing will occur in `QPC_solve`. If `stop_print` is negative, printing will occur once it has been started by `start_print`. The default is `stop_print = -1`.

`infeas_max` is a scalar variable of type default `INTEGER`, that specifies the number of iterations for which the overall infeasibility of the problem is not reduced by at least a factor `reduce_infeas` before the problem is flagged as infeasible (see `reduce_infeas`). The default is `infeas_max = 200`.

`muzero_fixed` is a scalar variable of type default `INTEGER`, that specifies the number of iterations before the initial barrier parameter (see `muzero`) may be altered. The default is `muzero_fixed = 1`.

`indicator_type` is a scalar variable of type default `INTEGER`, that specifies the type of indicator used to assess when a variable or constraint is active. Possible values are:

- 1 a variable/constraint is active if and only if the distance to its nearest bound is no larger than `indicator_tol_p` (see below).
- 2 a variable/constraint is active if and only if the distance to its nearest bound is no larger than `indicator_tol_pd` (see below) times the magnitude of its corresponding dual variable.
- 3 a variable/constraint is active if and only if the distance to its nearest bound is no larger than `indicator_tol_tapia` (see below) times the distance to the same bound at the previous iteration.

The default is `indicator_type = 3`.

`arc` is a scalar variable of type default `INTEGER`, that specifies the type of residual trajectory used to define the path to the solution. Possible values are:

- 1 the residual trajectory proposed by Zhang will be used.
- 2 the residual trajectory proposed by Zhao and Sun will be used; note this trajectory does not necessarily ensure convergence, so should be used with caution.
- 3 a combination in which Zhang's trajectory is used until the method determines that Zhou and Sun's will be better.
- 4 a mixed linear-quadratic variant of Zhang's proposal will be used.

The default is `arc = 1`.

`series_order` is a scalar variable of type default `INTEGER`, that specifies the order of (Puiseux or Taylor) series to approximate the residual trajectory. The default is `series_order = 2`.

`infinity` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that is used to specify which constraint bounds are infinite. Any bound larger than `infinity` in modulus will be regarded as infinite. The default is `infinity = 1019`.

`stop_abs_p` and `stop_rel_p` are scalar variables of type default `REAL` (double precision in `GALAHAD_QPC_double`), that hold the required absolute and relative accuracy for the primal infeasibility (see Section 4). The absolute value of each component of the primal infeasibility on exit is required to be smaller than the larger of `stop_abs_p` and `stop_rel_p` times a "typical value" for this component. The defaults are `stop_abs_p = stop_rel_p =  $u^{1/3}$` , where `u` is `EPSILON(1.0)` (`EPSILON(1.0D0)` in `GALAHAD_QPC_double`).

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`stop_abs_d` and `stop_rel_d` are scalar variables of type default REAL (double precision in GALAHAD\_QPC\_double), that hold the required absolute and relative accuracy for the dual infeasibility (see Section 4). The absolute value of each component of the dual infeasibility on exit is required to be smaller than the larger of `stop_abs_p` and `stop_rel_p` times a “typical value” for this component. The defaults are `stop_abs_d = stop_rel_d =  $u^{1/3}$` , where  $u$  is EPSILON(1.0) (EPSILON(1.0D0) in GALAHAD\_QPC\_double).

`stop_abs_c` and `stop_rel_c` are scalar variables of type default REAL (double precision in GALAHAD\_QPC\_double), that hold the required absolute and relative accuracy for the violation of complementary slackness (see Section 4). The absolute value of each component of the complementary slackness on exit is required to be smaller than the larger of `stop_abs_p` and `stop_rel_p` times a “typical value” for this component. The defaults are `stop_abs_c = stop_rel_c =  $u^{1/3}$` , where  $u$  is EPSILON(1.0) (EPSILON(1.0D0) in GALAHAD\_QPC\_double).

`perturb_h` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies any perturbation that is to be added to the diagonal of  $\mathbf{H}$ . The default is `perturb_h = 0.0`.

`prfeas` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that aims to specify the closest that any initial variable may be to infeasibility. Any variable closer to infeasibility than `prfeas` will be moved to `prfeas` from the offending bound. However, if a variable is range bounded, and its bounds are closer than `prfeas` apart, it will be moved to the mid-point of the two bounds. The default is `prfeas = 104`.

`dufeas` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that aims to specify the closest that any initial dual variable or Lagrange multiplier may be to infeasibility. Any variable closer to infeasibility than `prfeas` will be moved to `dufeas` from the offending bound. However, if a dual variable is range bounded, and its bounds are closer than `dufeas` apart, it will be moved to the mid-point of the two bounds. The default is `dufeas = 104`.

`muzero` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the initial value of the barrier parameter. If `muzero` is not positive, it will be reset automatically to an appropriate value. The default is `muzero = -1.0`.

`tau` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the weight attached to primal-dual infeasibility compared to complementarity when assessing step acceptance. The default is `tau = 1.0`.

`gamma_c` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the smallest value that individual complementarity pairs are allowed to be relative to the average as the iteration proceeds. The default is `gamma_c = 10-5`.

`gamma_f` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the smallest value the average complementarity is allowed to be relative to the primal-dual infeasibility as the iteration proceeds. The default is `gamma_c = 10-5`.

`reduce_infeas` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies the least factor by which the overall infeasibility of the problem must be reduced, over `infeas_max` consecutive iterations, for it not be declared infeasible (see `infeas_max`). The default is `reduce_infeas = 0.99`.

`obj_unbounded` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies smallest value of the objective function that will be tolerated before the problem is declared to be unbounded from below. The default is `obj_unbounded =  $-u^{-2}$` , where  $u$  is EPSILON(1.0) (EPSILON(1.0D0) in GALAHAD\_QPC\_double).

`potential_unbounded` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies smallest value of the potential function divided by the number of one-sided variable and constraint bounds that will be tolerated before the analytic center is declared to be unbounded. The default is `potential_unbounded = -10.0`.

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`identical_bounds_tol` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double). Every pair of constraint bounds  $(c_i^l, c_i^u)$  or  $(x_j^l, x_j^u)$  that is closer than `identical_bounds_tol` will be reset to the average of their values,  $\frac{1}{2}(c_i^l + c_i^u)$  or  $\frac{1}{2}(x_j^l + x_j^u)$  respectively. The default is `identical_bounds_tol = u`, where  $u$  is EPSILON(1.0) (EPSILON(1.0D0) in GALAHAD\_QPC\_double).

`indicator_tol_p` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double) that provides the indicator tolerance associated with the test `indicator_type = 1`. The default is `indicator_tol_p = u1/3`, where  $u$  is EPSILON(1.0) (EPSILON(1.0D0) in GALAHAD\_QPC\_double).

`indicator_tol_pd` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double) that provides the indicator tolerance associated with the test `indicator_type = 2`. The default is `indicator_tol_pd = 1.0`.

`indicator_tol_tapia` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double) that provides the indicator tolerance associated with the test `indicator_type = 3`. The default is `indicator_tol_tapia = 0.9`.

`cpu_time_limit` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that is used to specify the maximum permitted CPU time. Any negative value indicates no limit will be imposed. The default is `cpu_time_limit = - 1.0`.

`clock_time_limit` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that is used to specify the maximum permitted elapsed system clock time. Any negative value indicates no limit will be imposed. The default is `clock_time_limit = - 1.0`.

`remove_dependencies` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the algorithm is to attempt to remove any linearly dependent constraints before solving the problem, and `.FALSE.` otherwise. We recommend removing linearly dependencies. The default is `remove_dependencies = .TRUE..`

`treat_zero_bounds_as_general` is a scalar variable of type default LOGICAL. If it is set to `.FALSE.`, variables which are only bounded on one side, and whose bound is zero, will be recognised as non-negativities/non-positivities rather than simply as lower- or upper-bounded variables. If it is set to `.TRUE.`, any variable bound  $x_j^l$  or  $x_j^u$  which has the value 0.0 will be treated as if it had a general value. Setting `treat_zero_bounds_as_general` to `.TRUE.` has the advantage that if a sequence of problems are reordered, then bounds which are “accidentally” zero will be considered to have the same structure as those which are nonzero. However, GALAHAD\_QPC is able to take special advantage of non-negativities/non-positivities, so if a single problem, or if a sequence of problems whose bound structure is known not to change, is/are to be solved, it will pay to set the variable to `.FALSE..` The default is `treat_zero_bounds_as_general = .FALSE..`

`just_feasible` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the algorithm should stop as soon as a feasible point of the constraint set is found, and `.FALSE.` otherwise. The default is `just_feasible = .FALSE..`

`getdua` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the user-provided estimates of the dual variables should be replaced by estimates whose aim is to try to balance the requirements of dual feasibility and complementary slackness, and `.FALSE.` if users estimates are to be used. The default is `getdua = .FALSE..`

`puiseux` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if a Puiseux series will be used when extrapolating along the central path and `.FALSE.` if a Taylor series is preferred. We recommend using the Puiseux series unless the solution is known to be non-degenerate. The default is `puiseux = .TRUE..`

`every_order` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if every order of approximation up to `series_order` will be tried and the best taken, and `.FALSE.` if only the exact order specified by `series_order` will be used. The default is `every_order = .TRUE..`

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`feasol` is a scalar variable of type default LOGICAL, that should be set `.TRUE.` if the final solution obtained will be perturbed so that variables close to their bounds are moved onto these bounds, and `.FALSE.` otherwise. The default is `feasol = .FALSE..`

`crossover` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the solution is to be defined in terms of linearly-independent constraints, and `.FALSE.` otherwise. The default is `crossover = .TRUE..`

`space_critical` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if space is critical when allocating arrays and `.FALSE.` otherwise. The package may run faster if `space_critical` is `.FALSE.` but at the possible expense of a larger storage requirement. The default is `space_critical = .FALSE..`

`deallocate_error_fatal` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the user wishes to terminate execution if a deallocation fails, and `.FALSE.` if an attempt to continue will be made. The default is `deallocate_error_fatal = .FALSE..`

`prefix` is a scalar variable of type default CHARACTER and length 30, that may be used to provide a user-selected character string to preface every line of printed output. Specifically, each line of output will be prefaced by the string `prefix(2:LEN(TRIM(prefix))-1)`, thus ignoring the first and last non-null components of the supplied string. If the user does not want to preface lines by such a string, they may use the default `prefix = ""`.

`FDC_control` is a scalar variable of type `FDC_control_type` whose components are used to control any detection of linear dependencies performed by the package `GALAHAD_FDC`. See the specification sheet for the package `GALAHAD_FDC` for details, and appropriate default values.

`SBLS_control` is a scalar variable of type `SBLS_control_type` whose components are used to control factorizations performed by the package `GALAHAD_SBLS`. See the specification sheet for the package `GALAHAD_SBLS` for details, and appropriate default values.

`FIT_control` is a scalar variable of type `FIT_control_type` whose components are used to control fitting of data to polynomials performed by the package `GALAHAD_FIT`. See the specification sheet for the package `GALAHAD_FIT` for details, and appropriate default values.

`ROOTS_control` is a scalar variable of type `ROOTS_control_type` whose components are used to control the polynomial root finding performed by the package `GALAHAD_ROOTS`. See the specification sheet for the package `GALAHAD_ROOTS` for details, and appropriate default values.

`CRO_control` is a scalar variable of type `CRO_control_type` whose components are used to control crossover performed by the package `GALAHAD_CRO`. See the specification sheet for the package `GALAHAD_CRO` for details, and appropriate default values.

#### 2.4.5 The derived data type for holding control parameters specifically for the working-set phase

The derived data type `QPA_control_type` is used to hold controlling data for the working-set phase of the calculation performed by the `GALAHAD` package `GALAHAD_QPA`. Components may be changed by calling `QPA_read_specfile` (see the documentation for `GALAHAD_QPA` for details). The components of `QPA_control_type` are:

`error` is a scalar variable of type default INTEGER, that holds the stream number for error messages. Printing of error messages in `QPA_solve` and `QPA_terminate` is suppressed if `error ≤ 0`. The default is `error = 6`.

`out` is a scalar variable of type default INTEGER, that holds the stream number for informational messages. Printing of informational messages in `QPA_solve` is suppressed if `out < 0`. The default is `out = 6`.

`print_level` is a scalar variable of type default INTEGER, that is used to control the amount of informational output which is required. No informational output will occur if `print_level ≤ 0`. If `print_level = 1`, a single line of output will be produced for each iteration of the process. If `print_level ≥ 2`, this output will be increased to provide significant detail of each iteration. The default is `print_level = 0`.

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`maxit` is a scalar variable of type default `INTEGER`, that holds the maximum number of iterations which will be allowed in `QPA_solve`. The default is `maxit = 1000`.

`start_print` is a scalar variable of type default `INTEGER`, that specifies the first iteration for which printing will occur in `QPA_solve`. If `start_print` is negative, printing will occur from the outset. The default is `start_print = -1`.

`stop_print` is a scalar variable of type default `INTEGER`, that specifies the last iteration for which printing will occur in `QPA_solve`. If `stop_print` is negative, printing will occur once it has been started by `start_print`. The default is `stop_print = -1`.

`factor` is a scalar variable of type default `INTEGER`, that indicates the type of factorization of the preconditioner to be used. Possible values are:

- 0 the type is chosen automatically on the basis of which option looks likely to be the most efficient.
- 1 a Schur-complement factorization will be used.
- 2 an augmented-system factorization will be used.

The default is `factor = 0`.

`max_col` is a scalar variable of type default `INTEGER`, that specifies the maximum number of nonzeros in a column of **A** which is permitted by the Schur-complement factorization. The default is `max_col = 35`.

`max_sc` is a scalar variable of type default `INTEGER`, that specifies the maximum number of columns permitted in the Schur complement of the reference matrix (see Section 4) before a refactorization is triggered. The default is `max_sc = 75`.

`itref_max` is a scalar variable of type default `INTEGER`, that specifies the maximum number of iterative refinements allowed with each application of the preconditioner. The default is `itref_max = 1`.

`cg_maxit` is a scalar variable of type default `INTEGER`, that holds the maximum number of conjugate-gradient inner iterations that may be performed during the computation of each search direction in `QPA_solve`. If `cg_itmax` is set to a negative number, it will be reset by `QPA_solve` to the dimension of the relevant linear system +1. The default is `cg_itmax = -1`.

`precon` is a scalar variable of type default `INTEGER`, that specifies which preconditioner to be used to accelerate the conjugate-gradient inner iteration. Possible values are:

- 0 the type is chosen automatically on the basis of which option looks likely to be the most efficient at any given stage of the solution process. Different preconditioners may be used at different stages.
- 1 a full factorization using the Hessian, which is equivalent to replacing the conjugate gradient inner iteration by a direct method. The Hessian may be perturbed to ensure that the resultant matrix is a preconditioner.
- 2 the Hessian matrix is replaced by the identity matrix.
- 3 the Hessian matrix is replaced by a band of given semi-bandwidth (see `nsemib` below).
- 4 the Hessian matrix terms in the current reference matrix (see Section 4) are replaced by the identity matrix.
- 5 the Hessian matrix terms outside a band of given semi-bandwidth in the current reference matrix are replaced by zeros (see `nsemib` below).

The default is `precon = 0`.

`nsemib` is a scalar variable of type default `INTEGER`, that specifies the semi-bandwidth of the band preconditioner when `precon = 3`, if appropriate. The default is `nsemib = 5`.

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`full_max_fill` is a scalar variable of type default `INTEGER`. If the ratio of the number of nonzeros in the factors of the reference matrix (see Section 4) to the number of nonzeros in the matrix itself exceeds `full_max_fill`, and the preconditioner is being selected automatically (`precon = 0`), a banded approximation (see `precon = 3`) will be used instead. The default is `full_max_fill = 10`.

`deletion_strategy` is a scalar variable of type default `INTEGER`, that specifies the rules used to determine which constraint to remove from the working set (see Section 4) when necessary to ensure further progress towards the solution. Possible values are:

- 0 the constraint whose Lagrange multiplier most violates its required optimality bound will be removed.
- 1 the most-recently added constraint whose Lagrange multiplier violates its required optimality bound will be removed.
- $k > 1$  among the  $k$  most-recently added constraints whose Lagrange multipliers violates their required optimality bounds, the one which most violates its bound will be removed.

The default is `deletion_strategy = 0`.

`restore_problem` is a scalar variable of type default `INTEGER`, that specifies how much of the input problem is to be retored on output. Possible values are:

- 0 nothing is restored.
- 1 the vector data  $\mathbf{g}$ ,  $\mathbf{c}^l$ ,  $\mathbf{c}^u$ ,  $\mathbf{x}^l$ , and  $\mathbf{x}^u$  will be restored to their input values.
- 2 the entire problem, that is the above vector data along with the Hessian matrix  $\mathbf{H}$  and the Jacobian matrix  $\mathbf{A}$ , will be restored.

The default is `restore_problem = 2`.

`monitor_residuals` is a scalar variable of type default `INTEGER`, that specifies the frequency at which working constraint residuals will be monitored to ensure feasibility. The residuals will be monitored every `monitor_residual` iterations. The default is `monitor_residuals = 1`.

`cold_start` is a scalar variable of type default `INTEGER`, that controls the initial working set (see Section 4)). Possible values are:

- 0 a "warm start" will be performed. The values set in `C_stat` and `B_stat` indicate which constraints will be included in the initial working set. (see `C_stat` and `B_stat` in Section 2.5.2).
- 1 the constraints "active" at `p%X` (see Section 2.5.2) will determine the initial working set.
- 2 the initial working set will be empty.
- 3 the initial working set will only contain equality constraints.
- 4 the initial working set will contain as many active constraints as possible, chosen (in order) from equality constraints, simple bounds, and finally general linear constraints.

The default is `cold_start = 3`.

`infeas_check_interval` is a scalar variable of type default `INTEGER`, that gives the number of iterations that are permitted before the infeasibility is checked for improvement. The default is `infeas_check_interval = 100`.

`infinity` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that is used to specify which constraint bounds are infinite. Any bound larger than `infinity` in modulus will be regarded as infinite. The default is `infinity = 1019`.

`feas_tol` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that specifies the maximum amount by which a constraint may be violated and yet still be considered to be satisfied. The default is `feas_tol =  $u^{3/4}$` , where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in `GALAHAD_QPC_double`).

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`obj_unbounded` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies smallest value of the objective function that will be tolerated before the problem is declared to be unbounded from below. The default is  $\text{potential\_unbounded} = -u^{-2}$ , where  $u$  is EPSILON(1.0) (EPSILON(1.0D0) in GALAHAD\_QPC\_double).

`increase_rho_g_factor` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that gives the factor by which the current penalty parameter  $\rho_g$  for the general constraints may be increased when solving quadratic programs. The default is `increase_rho_g_factor = 2`.

`increase_rho_b_factor` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that gives the factor by which the current penalty parameter  $\rho_b$  for the simple bound constraints may be increased when solving quadratic programs. The default is `increase_rho_b_factor = 2`.

`infeas_g_improved_by_factor` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies the relative improvement in the infeasibility that must be achieved when solving quadratic programs if the current value of  $\rho_g$  is to be maintained. Specifically if the infeasibility of the general constraints has not fallen by at least a factor `infeas_g_improved_by_factor` during the previous `infeas_check_interval` iterations, the penalty parameter will be increased by a factor `increase_rho_g_factor`. The default is `infeas_improved_g_by_factor = 0.75`.

`infeas_b_improved_by_factor` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that specifies the relative improvement in the infeasibility that must be achieved when solving quadratic programs if the current value of  $\rho_b$  is to be maintained. Specifically if the infeasibility of the simple bound constraints has not fallen by at least a factor `infeas_b_improved_by_factor` during the previous `infeas_check_interval` iterations, the penalty parameter will be increased by a factor `increase_rho_b_factor`. The default is `infeas_improved_b_by_factor = 0.75`.

`pivot_tol_for_dependencies` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the relative threshold pivot tolerance used by the matrix factorization when attempting to detect linearly dependent constraints. A value larger than `pivot_tol` is appropriate. See the documentation for the package SLS for details. The default is `pivot_tol_for_dependencies = 0.5`.

`multiplier_tol` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double). Any dual variable or Lagrange multiplier which is less than `multiplier_tol` outside its optimal interval will be regarded as being acceptable when checking for optimality. The default is  $\text{zero\_pivot} = \sqrt{u}$ , where  $u$  is EPSILON(1.0) (EPSILON(1.0D0) in GALAHAD\_QPC\_double).

`inner_stop_relative` and `inner_stop_absolute` are scalar variables of type default REAL (double precision in GALAHAD\_QPC\_double), that hold the relative and absolute convergence tolerances for the inner iteration (search direction) problem. and correspond to the values `control%stop_relative` and `control%stop_absolute` in the GALAHAD package GALAHAD\_GLTR. The defaults are `inner_stop_relative = 0.0` and `inner_stop_absolute =  $\sqrt{u}$` , where  $u$  is EPSILON(1.0) (EPSILON(1.0D0) in GALAHAD\_QPC\_double).

`cpu_time_limit` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that is used to specify the maximum permitted CPU time. Any negative value indicates no limit will be imposed. The default is `cpu_time_limit = - 1.0`.

`clock_time_limit` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that is used to specify the maximum permitted elapsed system clock time. Any negative value indicates no limit will be imposed. The default is `clock_time_limit = - 1.0`.

`treat_zero_bounds_as_general` is a scalar variable of type default LOGICAL. If it is set to `.FALSE.`, variables which are only bounded on one side, and whose bound is zero, will be recognised as non-negativities/non-positivities rather than simply as lower- or upper-bounded variables. If it is set to `.TRUE.`, any variable bound  $x_j^l$  or  $x_j^u$

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which has the value 0.0 will be treated as if it had a general value. Setting `treat_zero_bounds_as_general` to `.TRUE.` has the advantage that if a sequence of problems are reordered, then bounds which are “accidentally” zero will be considered to have the same structure as those which are nonzero. However, `GALAHAD_QPC` is able to take special advantage of non-negativities/non-positivities, so if a single problem, or if a sequence of problems whose bound structure is known not to change, is/are to be solved, it will pay to set the variable to `.FALSE.`. The default is `treat_zero_bounds_as_general = .FALSE.`

`randomize` is a scalar variable of type default `LOGICAL`, that must be set `.TRUE.` if the user wishes to perturb the constraint bounds by small random quantities during the first stage of the solution process, and `.FALSE.` otherwise. Any randomization will ultimately be removed. Randomization helps when solving degenerate problems and is usually to be recommended. The default is `randomize = .TRUE.`

`symmetric_linear_solver` is a scalar variable of type default `CHARACTER` and length 30, that specifies the external package to be used to solve any symmetric linear system that might arise. Current possible choices are `'sils'`, `'ma27'`, `'ma57'`, `'ma77'`, `'ma86'`, `'ma97'`, `ssids`, `'pardiso'` and `'wsmp'`, although only `'sils'` and, for OMP 4.0-compliant compilers, `'ssids'` are installed by default. See the documentation for the GALAHAD package SLS for further details. The default is `symmetric_linear_solver = 'sils'`.

`definite_linear_solver` is a scalar variable of type default `CHARACTER` and length 30, that specifies the external package to be used to solve any symmetric positive-definite linear system that might arise. Current possible choices are `'sils'`, `'ma27'`, `'ma57'`, `'ma77'`, `'ma86'`, `'ma87'`, `'ma97'`, `ssids`, `'pardiso'` and `'wsmp'`, although only `'sils'` and, for OMP 4.0-compliant compilers, `'ssids'` are installed by default. See the documentation for the GALAHAD package SLS for further details. The default is `definite_linear_solver = 'sils'`.

`prefix` is a scalar variable of type default `CHARACTER` and length 30, that may be used to provide a user-selected character string to preface every line of printed output. Specifically, each line of output will be prefaced by the string `prefix(2:LEN(TRIM(prefix))-1)`, thus ignoring the first and last non-null components of the supplied string. If the user does not want to preface lines by such a string, they may use the default `prefix = ""`.

`SLS_control` is a scalar variable argument of type `SLS_control_type` that is used to pass control options to external packages used to factorize relevant symmetric matrices that arise. See the documentation for the GALAHAD package SLS for further details. In particular, default values are as for SLS, except that `SLS_control%relative_pivot_tolerance` is reset to `pivot_tol`.

#### 2.4.6 The derived data type for holding control parameters specifically for the equality-constrained phase

The derived data type `EQP_control_type` is used to hold controlling data for the equality-constrained QP phase of the calculation (if any), performed by the GALAHAD package `GALAHAD_EQP`. Components may be changed by calling `EQP_read_specfile` (see the documentation for `GALAHAD_EQP` for details). The components of `EQP_control_type` are:

`error` is a scalar variable of type default `INTEGER`, that holds the stream number for error messages. Printing of error messages in `QPC_solve` and `QPC_terminate` is suppressed if `error ≤ 0`. The default is `error = 6`.

`out` is a scalar variable of type default `INTEGER`, that holds the stream number for informational messages. Printing of informational messages in `QPC_solve` is suppressed if `out < 0`. The default is `out = 6`.

`print_level` is a scalar variable of type default `INTEGER`, that is used to control the amount of informational output which is required. No informational output will occur if `print_level ≤ 0`. If `print_level = 1`, a single line of output will be produced for each iteration of the process. If `print_level ≥ 2`, this output will be increased to provide significant detail of each iteration. The default is `print_level = 0`.

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`new_h` is a scalar variable of type default `INTEGER`, that is used to indicate how **H** has changed (if at all) since the previous call to `QPC_solve`. Possible values are:

- 0 **H** is unchanged
- 1 the values in **H** have changed, but its nonzero structure is as before.
- 2 both the values and structure of **H** have changed.

The default is `new_h = 2`.

`new_a` is a scalar variable of type default `INTEGER`, that is used to indicate how **A** has changed (if at all) since the previous call to `QPC_solve`. Possible values are:

- 0 **A** is unchanged
- 1 the values in **A** have changed, but its nonzero structure is as before.
- 2 both the values and structure of **A** have changed.

The default is `new_a = 2`.

`cg_maxit` is a scalar variable of type default `INTEGER`, that is used to limit the number of conjugate-gradient iterations performed in the optimality phase. If `cg_maxit` is negative, no limit will be imposed. The default is `cg_maxit = 200`.

`radius` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that may be used to specify an upper bound on the norm of the allowed solution (a “trust-region” constraint) during the iterative solution of the optimality phase of the problem. This is particularly useful if the problem is unbounded from below. If `radius` is set too small, there is a possibility that this will preclude the package from finding the actual solution. If `initial_radius` is not positive, it will be reset to the default value, `initial_radius = SQRT(0.1*HUGE(1.0)) (SQRT(0.1*HUGE(1.0D0))` in `GALAHAD_QPC_double`).

`inner_stop_relative` and `inner_stop_absolute` are scalar variables of type default `REAL` (double precision in `GALAHAD_QPC_double`), that hold the relative and absolute convergence tolerances for the iterative solution of the optimality phase of the problem using the package `GALAHAD_GLTR`, and correspond to the values `control%stop_relative` and `control%stop_absolute` in that package. The defaults are `inner_stop_relative = 0.01` and `inner_stop_absolute =  $\sqrt{u}$` , where `u` is `EPSILON(1.0)` (`EPSILON(1.0D0)` in `GALAHAD_QPC_double`).

`max_infeasibility_relative` and `max_infeasibility_absolute` are scalar variables of type default `REAL` (double precision in `GALAHAD_QPC_double`), that hold the relative and absolute tolerances for assessing infeasibility in the feasibility phase. If the constraints are believed to be rank deficient and the norm of the residual  $\mathbf{Ax}_T + \mathbf{c}$  at a “typical” feasible point is larger than `max(max_infeasibility_relative*||A||, max_infeasibility_absolute)`, the problem will be marked as infeasible. The defaults are `max_infeasibility_relative = max_infeasibility_absolute =  $u^{0.75}$`  where `u` is `EPSILON(1.0)` (`EPSILON(1.0D0)` in `GALAHAD_QPC_double`).

`remove_dependencies` is a scalar variable of type default `LOGICAL`, that must be set `.TRUE.` if linear dependent constraints  $\mathbf{Ax} + \mathbf{c} = \mathbf{0}$  should be removed and `.FALSE.` otherwise. The default is `remove_dependencies = .TRUE..`

`space_critical` is a scalar variable of type default `LOGICAL`, that must be set `.TRUE.` if space is critical when allocating arrays and `.FALSE.` otherwise. The package may run faster if `space_critical` is `.FALSE.` but at the possible expense of a larger storage requirement. The default is `space_critical = .FALSE..`

`deallocate_error_fatal` is a scalar variable of type default `LOGICAL`, that must be set `.TRUE.` if the user wishes to terminate execution if a deallocation fails, and `.FALSE.` if an attempt to continue will be made. The default is `deallocate_error_fatal = .FALSE..`

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`FDC_control` is a scalar variable of type `FDC_control_type` whose components are used to control any detection of linear dependencies performed by the package `GALAHAD_FDC`. See the specification sheet for the package `GALAHAD_FDC` for details, and appropriate default values.

`GLTR_control` is a scalar variable argument of type `GLTR_control_type` that is used to pass control options to the conjugate-gradient solver used to solve linear systems that arise. See the documentation for the `GALAHAD` package `GLTR` for further details. In particular, default values are as for `GLTR`.

`SBLS_control` is a scalar variable argument of type `SBLS_control_type` that is used to pass control options to the symmetric block linear equation preconditioner used to help solve linear systems that arise. See the documentation for the `GALAHAD` package `SBLS` for further details. In particular, default values are as for `SBLS`.

`prefix` is a scalar variable of type default `CHARACTER` and length 30, that may be used to provide a user-selected character string to preface every line of printed output. Specifically, each line of output will be prefaced by the string `prefix(2:LEN(TRIM( prefix ))-1)`, thus ignoring the first and last non-null components of the supplied string. If the user does not want to preface lines by such a string, they may use the default `prefix = ""`.

#### 2.4.7 The derived data type for holding control parameters

The derived data type `QPC_control_type` is used to hold controlling data. Default values may be obtained by calling `QPC_initialize` (see Section 2.5.1), while components may also be changed by calling `QPC_read_specfile` (see Section 2.7.1). The components of `QPC_control_type` are:

`error` is a scalar variable of type default `INTEGER`, that holds the stream number for error messages. Printing of error messages in `QPC_solve` and `QPC_terminate` is suppressed if `error ≤ 0`. The default is `error = 6`.

`out` is a scalar variable of type default `INTEGER`, that holds the stream number for informational messages. Printing of informational messages in `QPC_solve` is suppressed if `out < 0`. The default is `out = 6`.

`print_level` is a scalar variable of type default `INTEGER`, that is used to control the amount of informational output which is required. No informational output will occur if `print_level ≤ 0`. If `print_level = 1`, a single line of output will be produced for each iteration of the process. If `print_level ≥ 2`, this output will be increased to provide significant detail of each iteration. The default is `print_level = 0`.

`restore_problem` is a scalar variable of type default `INTEGER`, that specifies how much of the input problem is to be retored on output. Possible values are:

- 0 nothing is restored.
- 1 the vector data  $\mathbf{g}$ ,  $\mathbf{c}^l$ ,  $\mathbf{c}^u$ ,  $\mathbf{x}^l$ , and  $\mathbf{x}^u$  will be restored to their input values.
- 2 the entire problem, that is the above vector data along with the Hessian matrix  $\mathbf{H}$  and the Jacobian matrix  $\mathbf{A}$ , will be restored.

The default is `restore_problem = 2`.

`infinity` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that is used to specify which constraint bounds are infinite. Any bound larger than `infinity` in modulus will be regarded as infinite. The default is `infinity = 1019`.

`identical_bounds_tol` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`). Every pair of constraint bounds  $(c_i^l, c_i^u)$  or  $(x_j^l, x_j^u)$  that is closer than `identical_bounds_tol` will be reset to the average of their values,  $\frac{1}{2}(c_i^l + c_i^u)$  or  $\frac{1}{2}(x_j^l + x_j^u)$  respectively. The default is `identical_bounds_tol = u`, where  $u$  is `EPSILON(1.0)` (`EPSILON(1.0D0)` in `GALAHAD_QPC_double`).

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`rho_g` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the initial values of the parameter,  $\rho_g$ , used to weight the infeasibility term  $v_g(\mathbf{x})$  for the working-set phase (see Section 4).

`rho_b` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that holds the initial values of the parameter,  $\rho_b$ , used to weight the infeasibility term  $v_b(\mathbf{x})$  for the working-set phase (see Section 4).

`on_bound_tol` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double) that specifies the furthest a variable or constraint may be from one of its bounds to be regarded as active. The default is `on_bound_tol = u`, where  $u$  is EPSILON(1.0) (EPSILON(1.0D0) in GALAHAD\_QPC\_double).

`cpu_time_limit` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that is used to specify the maximum permitted CPU time. Any negative value indicates no limit will be imposed. The default is `cpu_time_limit = - 1.0`.

`clock_time_limit` is a scalar variable of type default REAL (double precision in GALAHAD\_QPC\_double), that is used to specify the maximum permitted elapsed system clock time. Any negative value indicates no limit will be imposed. The default is `clock_time_limit = - 1.0`.

`convex` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the problem is convex (i.e., if the matrix  $\mathbf{H}$  is positive semi-definite) and the user wishes to take advantage of the fact, and `.FALSE.` otherwise. The package will usually run faster for convex problems if `convex` is `.TRUE.`, but `convex` must be set `.FALSE.` if the user is uncertain as otherwise the package may fail. The default is `convex = .FALSE.`

`treat_zero_bounds_as_general` is a scalar variable of type default LOGICAL. If it is set to `.FALSE.`, variables which are only bounded on one side, and whose bound is zero, will be recognised as non-negativities/non-positivities rather than simply as lower- or upper-bounded variables. If it is set to `.TRUE.`, any variable bound  $x_j^l$  or  $x_j^u$  which has the value 0.0 will be treated as if it had a general value. Setting `treat_zero_bounds_as_general` to `.TRUE.` has the advantage that if a sequence of problems are reordered, then bounds which are “accidentally” zero will be considered to have the same structure as those which are nonzero. However, GALAHAD\_QPC is able to take special advantage of non-negativities/non-positivities, so if a single problem, or if a sequence of problems whose bound structure is known not to change, is/are to be solved, it will pay to set the variable to `.FALSE.`. The default is `treat_zero_bounds_as_general = .FALSE.`

`space_critical` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if space is critical when allocating arrays and `.FALSE.` otherwise. The package may run faster if `space_critical` is `.FALSE.` but at the possible expense of a larger storage requirement. The default is `space_critical = .FALSE.`

`deallocate_error_fatal` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the user wishes to terminate execution if a deallocation fails, and `.FALSE.` if an attempt to continue will be made. The default is `deallocate_error_fatal = .FALSE.`

`no_qpa` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the user wishes to skip the working-set phase of the computation, and `.FALSE.` if this phase is required. The default is `no_qpa = .FALSE.`

`no_qpb` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the user wishes to skip the interior-point phase of the computation, and `.FALSE.` if this phase is required. The default is `no_qpb = .FALSE.`

`qpb_or_qpa` is a scalar variable of type default LOGICAL, that must be set `.TRUE.` if the user wishes to use the interior-point phase of the computation, but only to follow this with the working-set phase if the former is unsuccessful, and to `.FALSE.` if both phases are to be used (subject to the requests made in `no_qpa` and `no_qpb`). The default is `qpb_or_qpa = .FALSE.`

`prefix` is a scalar variable of type default CHARACTER and length 30, that may be used to provide a user-selected character string to preface every line of printed output. Specifically, each line of output will be prefaced by the string `prefix(2:LEN(TRIM(prefix))-1)`, thus ignoring the first and last non-null components of the supplied string. If the user does not want to preface lines by such a string, they may use the default `prefix = ""`.

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`QPA_control` is a scalar variable of type `QPA_control_type` (see Section 2.4.5 whose components are used to control the working-set phase of the calculation, performed by the package `GALAHAD_QPA`).

`QPB_control` is a scalar variable of type `QPB_control_type` (see Section 2.4.3 whose components are used to control the interior-point phase of the calculation, performed by the package `GALAHAD_QPB`).

`CQP_control` is a scalar variable of type `CQP_control_type` (see Section 2.4.4 whose components are used to control the convex primal-dual phase of the calculation, performed by the package `GALAHAD_CQP`).

`EQP_control` is a scalar variable of type `EQP_control_type` (see Section 2.4.6 whose components are used to control the equality-constrained phase of the calculation (if any), performed by the package `GALAHAD_EQP`).

`FDC_control` is a scalar variable of type `FDC_control_type` whose components are used to control any detection of linear dependencies performed by the package `GALAHAD_FDC`. See the specification sheet for the package `GALAHAD_FDC` for details, and appropriate default values.

### 2.4.8 The derived data type for holding timing information

The derived data type `QPC_time_type` is used to hold elapsed CPU and system clock times for the various parts of the calculation. The components of `QPC_time_type` are:

`total` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the total CPU time spent in the package.

`preprocess` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the CPU time spent reordering the problem to standard form prior to solution.

`find_dependent` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the CPU time spent detecting and removing linearly-dependent equality constraints

`analyse` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the CPU time spent analysing the required matrices prior to factorization.

`factorize` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the CPU time spent factorizing the required matrices.

`solve` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the CPU time spent computing the search direction.

`clock_total` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the total elapsed system clock time spent in the package.

`clock_preprocess` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the elapsed system clock time spent reordering the problem to standard form prior to solution.

`clock_find_dependent` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the elapsed system clock time spent detecting and removing linearly-dependent equality constraints

`clock_analyse` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the elapsed system clock time spent analysing the required matrices prior to factorization.

`clock_factorize` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the elapsed system clock time spent factorizing the required matrices.

`clock_solve` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that gives the elapsed system clock time spent computing the search direction.

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#### 2.4.9 The derived data type for holding informational parameters specifically for the interior-point phase

The derived data type `QPB_inform_type` is used to hold parameters that give information about the progress of the working-set phase of the algorithm performed by the GALAHAD package `GALAHAD_QPB`. The components of `QPB_inform_type` are:

`status` is a scalar variable of type default `INTEGER`, that gives the exit status of the algorithm. See Section 2.6 for details.

`alloc_status` is a scalar variable of type default `INTEGER`, that gives the status of the last attempted array allocation or deallocation. This will be 0 if `status = 0`.

`bad_alloc` is a scalar variable of type default `CHARACTER` and length 80, that gives the name of the last internal array for which there were allocation or deallocation errors. This will be the null string if `status = 0`.

`iter` is a scalar variable of type default `INTEGER`, that gives the total number of iterations required.

`factorization_status` is a scalar variable of type default `INTEGER`, that gives the return status from the matrix factorization.

`factorization_integer` is a scalar variable of type long `INTEGER`, that gives the amount of integer storage used for the matrix factorization.

`factorization_real` is a scalar variable of type long `INTEGER`, that gives the amount of real storage used for the matrix factorization.

`nfacts` is a scalar variable of type default `INTEGER`, that gives the total number of factorizations performed.

`nbacts` is a scalar variable of type default `INTEGER`, that gives the total number of backtracks performed during the sequence of line searches.

`nmods` is a scalar variable of type default `INTEGER`, that gives the total number of factorizations which were modified to ensure that the matrix is an appropriate preconditioner.

`obj` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the value of the objective function at the best estimate of the solution found.

`non_negligible_pivot` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the value of the smallest pivot larger than `control%zero_pivot` when searching for dependent linear constraints. If `non_negligible_pivot` is close to `control%zero_pivot`, this may indicate that there are further dependent constraints, and it may be worth increasing `control%zero_pivot` above `non_negligible_pivot` and solving again.

`feasible` is a scalar variable of type default `LOGICAL`, that has the value `.TRUE.` if the output value of `x` satisfies the constraints, and the value `.FALSE.` otherwise.

`time` is a scalar variable of type `QPC_time_type` whose components are used to hold elapsed CPU and system clock times for the various parts of the calculation (see Section 2.4.8).

`LSQP_inform` is a scalar variable of type `LSQP_inform_type` whose components are used to provide information about the initial feasible point calculation performed by the package `GALAHAD_LSQP`. See the specification sheet for the package `GALAHAD_LSQP` for details, and appropriate default values.

`FDC_inform` is a scalar variable of type `FDC_inform_type` whose components are used to provide information about any detection of linear dependencies performed by the package `GALAHAD_FDC`. See the specification sheet for the package `GALAHAD_FDC` for details.

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`SBLS_inform` is a scalar variable of type `SBLS_inform_type` whose components are used to provide information about factorizations performed by the package `GALAHAD_SBLS`. See the specification sheet for the package `GALAHAD_SBLS` for details.

`GLTR_inform` is a scalar variable of type `GLTR_inform_type` whose components are used to provide information about the step calculation performed by the package `GALAHAD_GLTR`. See the specification sheet for the package `GALAHAD_GLTR` for details.

#### 2.4.10 The derived data type for holding informational parameters specifically for the interior-point phase in the convex case

The derived data type `CQP_inform_type` is used to hold parameters that give information about the progress of the working-set phase of the algorithm performed by the `GALAHAD` package `GALAHAD_CQP`. The components of `CQP_inform_type` are:

`status` is a scalar variable of type default `INTEGER`, that gives the exit status of the algorithm. See Section 2.6 for details.

`alloc_status` is a scalar variable of type default `INTEGER`, that gives the status of the last attempted array allocation or deallocation. This will be 0 if `status = 0`.

`bad_alloc` is a scalar variable of type default `CHARACTER` and length 80, that gives the name of the last internal array for which there were allocation or deallocation errors. This will be the null string if `status = 0`.

`factorization_status` is a scalar variable of type default `INTEGER`, that gives the return status from the matrix factorization.

`factorization_integer` is a scalar variable of type long `INTEGER`, that gives the amount of integer storage used for the matrix factorization.

`factorization_real` is a scalar variable of type long `INTEGER`, that gives the amount of real storage used for the matrix factorization.

`nfacts` is a scalar variable of type default `INTEGER`, that gives the total number of factorizations performed.

`nbacts` is a scalar variable of type default `INTEGER`, that gives the total number of backtracks performed during the sequence of line searches.

`threads` is a scalar variable of type default `INTEGER`, that gives the total number of threads used for parallel execution.

`iter` is a scalar variable of type default `INTEGER`, that gives the number of iterations performed.

`obj` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the value of the objective function at the best estimate of the solution found.

`primal_infeasibility` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the norm of the violation of primal optimality (see Section 2.4.8) at the best estimate of the solution found.

`dual_infeasibility` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the norm of the violation of dual optimality (see Section 2.4.8) at the best estimate of the solution found.

`complementary_slackness` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the norm of the violation of complementary slackness (see Section 2.4.8) at the best estimate of the solution found.

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`feasible` is a scalar variable of type `default LOGICAL`, that has the value `.TRUE.` if the output value of `x` satisfies the constraints, and the value `.FALSE.` otherwise.

`time` is a scalar variable of type `QPC_time_type` whose components are used to hold elapsed CPU and system clock times for the various parts of the calculation (see Section 2.4.8).

`FDC_inform` is a scalar variable of type `FDC_inform_type` whose components are used to provide information about any detection of linear dependencies performed by the package `GALAHAD_FDC`. See the specification sheet for the package `GALAHAD_FDC` for details.

`SBLS_inform` is a scalar variable of type `SBLS_inform_type` whose components are used to provide information about factorizations performed by the package `GALAHAD_SBLS`. See the specification sheet for the package `GALAHAD_SBLS` for details.

`FIT_inform` is a scalar variable of type `FIT_inform_type` whose components are used to provide information about the fitting of data to polynomials performed by the package `GALAHAD_FIT`. See the specification sheet for the package `GALAHAD_FIT` for details.

`ROOTS_inform` is a scalar variable of type `ROOTS_inform_type` whose components are used to provide information about the polynomial root finding performed by the package `GALAHAD_ROOTS`. See the specification sheet for the package `GALAHAD_ROOTS` for details.

`CRO_inform` is a scalar variable of type `CRO_inform_type` whose components are used to provide information about the crossover performed by the package `GALAHAD_CRO`. See the specification sheet for the package `GALAHAD_CRO` for details.

#### 2.4.11 The derived data type for holding informational parameters specifically from the working-set phase

The derived data type `QPA_inform_type` is used to hold parameters that give information about the progress of the working-set phase of the algorithm performed by the `GALAHAD` package `GALAHAD_QPA`. The components of `QPA_inform_type` are:

`status` is a scalar variable of type `default INTEGER`, that gives the exit status of the algorithm. See Section 2.6 for details.

`alloc_status` is a scalar variable of type `default INTEGER`, that gives the status of the last attempted array allocation or deallocation. This will be 0 if `status = 0`.

`bad_alloc` is a scalar variable of type `default CHARACTER` and length 80, that gives the name of the last internal array for which there were allocation or deallocation errors. This will be the null string if `status = 0`.

`major_iter` is a scalar variable of type `default INTEGER`, that gives the total number of major iterations required.

`iter` is a scalar variable of type `default INTEGER`, that gives the total number of iterations required.

`cg_iter` is a scalar variable of type `default INTEGER`, that gives the total number of conjugate-gradient inner iterations required.

`factorization_status` is a scalar variable of type `default INTEGER`, that gives the return status from the matrix factorization.

`factorization_integer` is a scalar variable of type `long INTEGER`, that gives the amount of integer storage used for the matrix factorization.

`factorization_real` is a scalar variable of type `long INTEGER`, that gives the amount of real storage used for the matrix factorization.

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`nfacts` is a scalar variable of type default `INTEGER`, that gives the total number of factorizations performed.

`nmods` is a scalar variable of type default `INTEGER`, that gives the total number of factorizations which were modified to ensure that the matrix is an appropriate preconditioner.

`num_g_infeas` is a scalar variable of type default `INTEGER`, that gives the total number of general constraints that are violated.

`num_b_infeas` is a scalar variable of type default `INTEGER`, that gives the total number of simple bound constraints that are violated.

`obj` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the value of the objective function at the best estimate of the solution found.

`infeas_g` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the value of the infeasibility  $v_g(\mathbf{x})$ .

`infeas_b` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the value of the infeasibility  $v_b(\mathbf{x})$ .

`merit` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the value of the merit function  $q(\mathbf{x}) + \rho_g v_g(\mathbf{x}) + \rho_b v_b(\mathbf{x})$  at the best estimate of the solution found.

`time` is a scalar variable of type `QPC_time_type` whose components are used to hold elapsed CPU and system clock times for the various parts of the calculation (see Section 2.4.8).

`SLS_inform` is a scalar variable argument of type `SLS_inform_type` that is used to pass information concerning the progress of the external packages used to factorize relevant symmetric matrices that arise. See the documentation for the GALAHAD package `SLS` for further details.

#### 2.4.12 The derived data type for holding informational parameters specifically for the equality-constrained phase

The derived data type `EQP_inform_type` is used to hold parameters that give information about the progress of the equality-constrained phase of the algorithm (if any) performed by the GALAHAD package `GALAHAD_EQP`. The components of `EQP_inform_type` are:

`status` is a scalar variable of type default `INTEGER`, that gives the exit status of the algorithm. See Section 2.6 for details.

`alloc_status` is a scalar variable of type default `INTEGER`, that gives the status of the last attempted array allocation or deallocation. This will be 0 if `status = 0`.

`bad_alloc` is a scalar variable of type default `CHARACTER` and length 80, that gives the name of the last internal array for which there were allocation or deallocation errors. This will be the null string if `status = 0`.

`cg_iter` is a scalar variable of type default `INTEGER`, that gives the total number of conjugate-gradient iterations required.

`factorization_integer` is a scalar variable of type long `INTEGER`, that gives the amount of integer storage used for the matrix factorization.

`factorization_real` is a scalar variable of type long `INTEGER`, that gives the amount of real storage used for the matrix factorization.

`obj` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the value of the objective function at the best estimate of the solution found.

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`time` is a scalar variable of type `QPC_time_type` whose components are used to hold elapsed CPU and system clock times for the various parts of the calculation (see Section 2.4.8).

`FDC_inform` is a scalar variable of type `FDC_inform_type` whose components are used to provide information about any detection of linear dependencies performed by the package `GALAHAD_FDC`. See the specification sheet for the package `GALAHAD_FDC` for details.

`SBLS_inform` is a scalar variable of type `SBLS_inform_type` whose components are used to hold information relating to the formation and factorization of the preconditioner. See the documentation for the `GALAHAD` package `SBLS` for further details.

`GLTR_inform` is a scalar variable of type `GLTR_inform_type` whose components are used to hold information relating to the computation of the solution via the conjugate-gradient method. See the documentation for the `GALAHAD` package `GLTR` for further details.

### 2.4.13 The derived data type for holding informational parameters

The derived data type `QPC_inform_type` is used to hold parameters that give information about the progress and needs of the algorithm. The components of `QPC_inform_type` are:

`status` is a scalar variable of type default `INTEGER`, that gives the exit status of the algorithm. See Section 2.6 for details.

`alloc_status` is a scalar variable of type default `INTEGER`, that gives the status of the last attempted array allocation or deallocation. This will be 0 if `status = 0`.

`bad_alloc` is a scalar variable of type default `CHARACTER` and length 80, that gives the name of the last internal array for which there were allocation or deallocation errors. This will be the null string if `status = 0`.

`factorization_status` is a scalar variable of type default `INTEGER`, that gives the return status from the matrix factorization.

`factorization_integer` is a scalar variable of type long `INTEGER`, that gives the amount of integer storage used for the matrix factorization.

`factorization_real` is a scalar variable of type long `INTEGER`, that gives the amount of real storage used for the matrix factorization.

`nfacts` is a scalar variable of type default `INTEGER`, that gives the total number of factorizations performed.

`nmods` is a scalar variable of type default `INTEGER`, that gives the total number of factorizations which were modified to ensure that the matrix is an appropriate preconditioner.

`obj` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the value of the objective function at the best estimate of the solution found.

`non_negligible_pivot` is a scalar variable of type default `REAL` (double precision in `GALAHAD_QPC_double`), that holds the value of the smallest pivot larger than `control%zero_pivot` when searching for dependent linear constraints. If `non_negligible_pivot` is close to `control%zero_pivot`, this may indicate that there are further dependent constraints, and it may be worth increasing `control%zero_pivot` above `non_negligible_pivot` and solving again.

`p_found` is a scalar variable of type default `LOGICAL`, that has the value `.TRUE.` if the active-set predictions have been computed, and the value `.FALSE.` otherwise.

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`time` is a scalar variable of type `QPC_time_type` whose components are used to hold elapsed CPU and system clock times for the various parts of the calculation (see Section 2.4.8).

`QPA_inform` is a scalar variable of type `QPA_inform_type` (see Section 2.4.11 whose components provide information about the progress and needs of the working-set phase of the calculation performed by the package `GALAHAD_QPA`).

`QPB_inform` is a scalar variable of type `QPB_inform_type` (see Section 2.4.9 whose components provide information about the progress and needs of the interior-point phase of the calculation performed by the package `GALAHAD_QPB`).

`CQP_inform` is a scalar variable of type `CQP_inform_type` (see Section 2.4.10 whose components provide information about the progress and needs of the primal-dual phase of the calculation performed by the package `GALAHAD_CQP`).

`EQP_inform` is a scalar variable of type `EQP_inform_type` (see Section 2.4.12 whose components provide information about the progress and needs of the equality-constrained phase of the calculation (if any) performed by the package `GALAHAD_EQP`).

`FDC_inform` is a scalar variable of type `FDC_inform_type` whose components are used to provide information about any detection of linear dependencies performed by the package `GALAHAD_FDC`. See the specification sheet for the package `GALAHAD_FDC` for details.

#### 2.4.14 The derived data type for holding problem data

The derived data type `QPC_data_type` is used to hold all the data for a particular problem, or sequences of problems with the same structure, between calls of `QPC` procedures. This data should be preserved, untouched, from the initial call to `QPC_initialize` to the final call to `QPC_terminate`.

### 2.5 Argument lists and calling sequences

There are three procedures for user calls (see Section 2.7 for further features):

1. The subroutine `QPC_initialize` is used to set default values, and initialize private data, before solving one or more problems with the same sparsity and bound structure.
2. The subroutine `QPC_solve` is called to solve the problem.
3. The subroutine `QPC_terminate` is provided to allow the user to automatically deallocate array components of the private data, allocated by `QPC_solve`, at the end of the solution process. It is important to do this if the data object is re-used for another problem **with a different structure** since `QPC_initialize` cannot test for this situation, and any existing associated targets will subsequently become unreachable.

We use square brackets [ ] to indicate OPTIONAL arguments.

#### 2.5.1 The initialization subroutine

Default values are provided as follows:

```
CALL QPC_initialize(data, control, inform)
```

`data` is a scalar `INTENT(INOUT)` argument of type `QPC_data_type` (see Section 2.4.14). It is used to hold data about the problem being solved.

`control` is a scalar `INTENT(OUT)` argument of type `QPC_control_type` (see Section 2.4.7). On exit, `control` contains default values for the components as described in Section 2.4.7. These values should only be changed after calling `QPC_initialize`.

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`inform` is a scalar `INTENT(OUT)` argument of type `QPC_inform_type` (see Section 2.4.13). A successful call to `QPC_initialize` is indicated when the component `status` has the value 0. For other return values of `status`, see Section 2.6.

## 2.5.2 The quadratic programming subroutine

The quadratic programming solution algorithm is called as follows:

```
CALL QPC_solve(p, C_stat, B_stat, data, control, inform)
```

`p` is a scalar `INTENT(INOUT)` argument of type `QPT_problem_type` (see Section 2.4.2). It is used to hold data about the problem being solved. For a new problem, the user must allocate all the array components, and set values for all components except `p%C`. `p%new_problem_structure` must be set `.TRUE.`, but will have been reset to `.FALSE.` on exit from `QPC_solve`. Users are free to choose whichever of the matrix formats described in Section 2.1 is appropriate for **A** and **H** for their application—different formats may be used for the two matrices.

For a problem with the same structure as one that has just been solved, the user may set `p%new_problem_structure` to `.FALSE.`, so long as `QPC_terminate` has not been called in the interim. The `INTEGER` components must be unaltered since the previous call to `QPC_solve`, but the `REAL` (double precision in `GALAHAD_QPC_double`) may be altered to reflect the new problem.

The components `p%X`, `p%Y` and `p%Z` must be set to initial estimates,  $\mathbf{x}^0$ , of the primal variables, **x**, Lagrange multipliers for the general constraints, **y**, and dual variables for the bound constraints, **z**, respectively. Inappropriate initial values will be altered, so the user should not be overly concerned if suitable values are not apparent, and may be content with merely setting `p%X=0.0`, `p%Y=0.0` and `p%Z=0.0`. The component `p%C` need not be set on entry.

On exit, the components `p%X`, `p%Y`, `p%Z` and `p%C` will contain the best estimates of the primal variables **x**, Lagrange multipliers for the general constraints **y**, dual variables for the bound constraints **z**, and values of the constraints **Ax** respectively. What of the remaining problem data has been restored depends upon the input value of the control parameter `control%restore_problem`. The return format for a restored array component will be the same as its input format. **Restrictions:**  $p%n > 0$ ,  $p%m \geq 0$ ,  $p%A\%ne \geq -2$  and  $p%H\%ne \geq -2$ .

`C_stat` is a rank-one `INTENT(INOUT)` array argument of dimension `p%m` and type default `INTEGER`, that indicates which of the general linear constraints are in the current working set. Possible values for `C_stat(i)`,  $i = 1, \dots, p%m$ , and their meanings are

- <0 the  $i$ -th general constraint is in the working set, on its lower bound,
- >0 the  $i$ -th general constraint is in the working set, on its upper bound, and
- 0 the  $i$ -th general constraint is not in the working set.

Suitable values must be supplied if `control%qa_control%cold_start = 0` on entry, but need not be provided for other input values of `control%cold_start`. Inappropriate values will be ignored. On exit, `C_stat` will contain values appropriate for the ultimate working set.

`B_stat` is a rank-one `INTENT(INOUT)` array argument of dimension `p%n` and type default `INTEGER`, that indicates which of the simple bound constraints are in the current working set. Possible values for `B_stat(j)`,  $j = 1, \dots, p%n$ , and their meanings are

- <0 the  $j$ -th simple bound constraint is in the working set, on its lower bound,
- >0 the  $j$ -th simple bound constraint is in the working set, on its upper bound, and
- 0 the  $j$ -th simple bound constraint is not in the working set.

Suitable values must be supplied if `control%qa_control%cold_start = 0` on entry, but need not be provided for other input values of `control%cold_start`. Inappropriate values will be ignored. On exit, `B_stat` will contain values appropriate for the ultimate working set.

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`data` is a scalar `INTENT(INOUT)` argument of type `QPC_data_type` (see Section 2.4.14). It is used to hold data about the problem being solved. It must not have been altered **by the user** since the last call to `QPC_initialize`.

`control` is a scalar `INTENT(IN)` argument of type `QPC_control_type` (see Section 2.4.7). Default values may be assigned by calling `QPC_initialize` prior to the first call to `QPC_solve`.

`inform` is a scalar `INTENT(INOUT)` argument of type `QPC_inform_type` (see Section 2.4.13). A successful call to `QPC_solve` is indicated when the component `status` has the value 0. For other return values of `status`, see Section 2.6.

### 2.5.3 The termination subroutine

All previously allocated arrays are deallocated as follows:

```
CALL QPC_terminate(data, control, inform)
```

`data` is a scalar `INTENT(INOUT)` argument of type `QPC_data_type` exactly as for `QPC_solve`, which must not have been altered **by the user** since the last call to `QPC_initialize`. On exit, array components will have been deallocated.

`control` is a scalar `INTENT(IN)` argument of type `QPC_control_type` exactly as for `QPC_solve`.

`inform` is a scalar `INTENT(OUT)` argument of type `QPC_inform_type` exactly as for `QPC_solve`. Only the component `status` will be set on exit, and a successful call to `QPC_terminate` is indicated when this component `status` has the value 0. For other return values of `status`, see Section 2.6.

## 2.6 Warning and error messages

A negative value of `inform%status` on exit from `QPC_solve` or `QPC_terminate` indicates that an error has occurred. No further calls should be made until the error has been corrected. Possible values are:

- 1. An allocation error occurred. A message indicating the offending array is written on unit `control%error`, and the returned allocation status and a string containing the name of the offending array are held in `inform%alloc_status` and `inform%bad_alloc` respectively.
- 2. A deallocation error occurred. A message indicating the offending array is written on unit `control%error` and the returned allocation status and a string containing the name of the offending array are held in `inform%alloc_status` and `inform%bad_alloc` respectively.
- 3. One of the restrictions `prob%n > 0` or `prob%m ≥ 0` or requirements that `prob%A_type` and `prob%H_type` contain its relevant string 'DENSE', 'COORDINATE', 'SPARSE\_BY\_ROWS' or 'DIAGONAL' has been violated.
- 4. The bound constraints are inconsistent.
- 5. The constraints appear to have no feasible point.
- 7. The objective function appears to be unbounded from below on the feasible set.
- 9. The analysis phase of the factorization failed; the return status from the factorization package is given in the component `inform%factor_status`.
- 10. The factorization failed; the return status from the factorization package is given in the component `inform%factor_status`.
- 16. The problem is so ill-conditioned that further progress is impossible.
- 17. The step is too small to make further impact.

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- 18. Too many iterations have been performed. This may happen if `control%maxit` is too small, but may also be symptomatic of a badly scaled problem.
- 19. The elapsed CPU or system clock time limit has been reached. This may happen if either `control%cpu_time_limit` or `control%clock_time_limit` is too small, but may also be symptomatic of a badly scaled problem.
- 23. An entry from the strict upper triangle of **H** has been specified.

## 2.7 Further features

In this section, we describe an alternative means of setting control parameters, that is components of the variable `control` of type `QPC_control_type` (see Section 2.4.7), by reading an appropriate data specification file using the subroutine `QPC_read_specfile`. This facility is useful as it allows a user to change QPC control parameters without editing and recompiling programs that call `QPC`.

A specification file, or `specfile`, is a data file containing a number of "specification commands". Each command occurs on a separate line, and comprises a "keyword", which is a string (in a close-to-natural language) used to identify a control parameter, and an (optional) "value", which defines the value to be assigned to the given control parameter. All keywords and values are case insensitive, keywords may be preceded by one or more blanks but values must not contain blanks, and each value must be separated from its keyword by at least one blank. Values must not contain more than 30 characters, and each line of the `specfile` is limited to 80 characters, including the blanks separating keyword and value.

The portion of the specification file used by `QPC_read_specfile` must start with a "BEGIN QPC" command and end with an "END" command. The syntax of the `specfile` is thus defined as follows:

```
(.. lines ignored by QPC_read_specfile ..)
 BEGIN QPC
 keyword value

 keyword value
 END
(.. lines ignored by QPC_read_specfile ..)
```

where `keyword` and `value` are two strings separated by (at least) one blank. The "BEGIN QPC" and "END" delimiter command lines may contain additional (trailing) strings so long as such strings are separated by one or more blanks, so that lines such as

```
BEGIN QPC SPECIFICATION
```

and

```
END QPC SPECIFICATION
```

are acceptable. Furthermore, between the "BEGIN QPC" and "END" delimiters, specification commands may occur in any order. Blank lines and lines whose first non-blank character is `!` or `*` are ignored. The content of a line after a `!` or `*` character is also ignored (as is the `!` or `*` character itself). This provides an easy manner to "comment out" some specification commands, or to comment specific values of certain control parameters.

The value of a control parameters may be of three different types, namely integer, logical or real. Integer and real values may be expressed in any relevant Fortran integer and floating-point formats (respectively). Permitted values for logical parameters are "ON", "TRUE", ".TRUE.", "T", "YES", "Y", or "OFF", "NO", "N", "FALSE", ".FALSE." and "F". Empty values are also allowed for logical control parameters, and are interpreted as "TRUE".

The specification file must be open for input when `QPC_read_specfile` is called, and the associated device number passed to the routine in device (see below). Note that the corresponding file is `REWINDed`, which makes it possible

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to combine the specifications for more than one program/routine. For the same reason, the file is not closed by `QPC_read_specfile`.

Control parameters corresponding to the components `QCP_control`, `QPA_control` and `QPB_control` may be changed by including additional sections enclosed by “BEGIN QCP” and “END QCP”, “BEGIN QPA” and “END QPA”, and “BEGIN QPB” and “END QPB”, respectively. See the specification sheets for the packages `GALAHAD_QCP`, `GALAHAD_QPA` and `GALAHAD_QPB` for further details.

### 2.7.1 To read control parameters from a specification file

Control parameters may be read from a file as follows:

```
CALL QPC_read_specfile(control, device)
```

`control` is a scalar `INTENT(INOUT)` argument of type `QPC_control_type` (see Section 2.4.7). Default values should have already been set, perhaps by calling `QPC_initialize`. On exit, individual components of `control` may have been changed according to the commands found in the specfile. Specfile commands and the component (see Section 2.4.7) of `control` that each affects are given in Table 2.1.

| command                                   | component of control                       | value type |
|-------------------------------------------|--------------------------------------------|------------|
| <code>error-printout-device</code>        | <code>%error</code>                        | integer    |
| <code>printout-device</code>              | <code>%out</code>                          | integer    |
| <code>print-level</code>                  | <code>%print_level</code>                  | integer    |
| <code>restore-problem-on-output</code>    | <code>%restore_problem</code>              | integer    |
| <code>infinity-value</code>               | <code>%infinity</code>                     | real       |
| <code>identical-bounds-tolerance</code>   | <code>%identical_bounds_tol</code>         | real       |
| <code>initial-rho-g</code>                | <code>%rho_g</code>                        | real       |
| <code>initial-rho-b</code>                | <code>%rho_b</code>                        | real       |
| <code>on-bound-tolerance</code>           | <code>%on_bound_tol</code>                 | real       |
| <code>maximum-cpu-time-limit</code>       | <code>%cpu_time_limit</code>               | real       |
| <code>maximum-clock-time-limit</code>     | <code>%clock_time_limit</code>             | real       |
| <code>convex</code>                       | <code>%convex</code>                       | logical    |
| <code>treat-zero-bounds-as-general</code> | <code>%treat_zero_bounds_as_general</code> | logical    |
| <code>space-critical</code>               | <code>%space_critical</code>               | logical    |
| <code>deallocate-error-fatal</code>       | <code>%deallocate_error_fatal</code>       | logical    |
| <code>no-qp-a-phase</code>                | <code>%no_qpa</code>                       | logical    |
| <code>no-qp-b-phase</code>                | <code>%no_qpb</code>                       | logical    |

Table 2.1: Specfile commands and associated components of `control`.

`device` is a scalar `INTENT(IN)` argument of type `default INTEGER`, that must be set to the unit number on which the specfile has been opened. If `device` is not open, `control` will not be altered and execution will continue, but an error message will be printed on unit `control%error`.

### 2.8 Information printed

If `control%print_level` is positive, information about the progress of the algorithm will be printed on unit `control%out`. If `control%print_level = 1`, a single line of output will be produced for each iteration of the process. For the initial-feasible-point phase, this will include values of the current primal and dual infeasibility, and violation of complementary slackness, the feasibility-phase objective value, the current steplength, the value of the barrier parameter, the number of backtracks in the linesearch and the elapsed CPU time in seconds. Once a suitable feasible

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point has been found, the iteration is divided into major iterations, at which the barrier parameter is reduced, and minor iterations, and which the barrier function is approximately minimized for the current value of the barrier parameter. For the major iterations, the value of the barrier parameter, the required values of dual feasibility and violation of complementary slackness, and the current constraint infeasibility are reported. Each minor iteration of the optimality phase results in a line giving the current dual feasibility and violation of complementary slackness, the objective function value, the ratio of predicted to achieved reduction of the objective function, the trust-region radius, the number of backtracks in the linesearch, the number of conjugate-gradient iterations taken, and the elapsed CPU time in seconds.

If `control%print_level`  $\geq 2$  this output will be increased to provide significant detail of each iteration. This extra output includes residuals of the linear systems solved, and, for larger values of `control%print_level`, values of the primal and dual variables and Lagrange multipliers.

### 3 GENERAL INFORMATION

**Use of common:** None.

**Workspace:** Provided automatically by the module.

**Other routines called directly:** None.

**Other modules used directly:** `QPC_solve` calls the GALAHAD packages `GALAHAD_CLOCK`, `GALAHAD_SYMBOLS`, `GALAHAD_SPACE`, `GALAHAD_NORMS`, `GALAHAD_SORT`, `GALAHAD_QPT`, `GALAHAD_QPP`, `GALAHAD_QPD`, `GALAHAD_EQP`, `GALAHAD_LSQP`, `GALAHAD_CQP`, `GALAHAD_QPA`, `GALAHAD_QPB`, `GALAHAD_SPECFILE` and `GALAHAD_FDC`.

**Input/output:** Output is under control of the arguments `control%error`, `control%out` and `control%print_level`.

**Restrictions:** `prob%n`  $> 0$ , `prob%m`  $\geq 0$ , `prob%A_type` and `prob%H_type`  $\in \{ 'DENSE', 'COORDINATE', 'SPARSE_BY_ROWS', 'DIAGONAL' \}$ .

**Portability:** ISO Fortran 95 + TR 15581 or Fortran 2003. The package is thread-safe.

### 4 METHOD

The required solution  $\mathbf{x}$  necessarily satisfies the primal optimality conditions

$$\mathbf{Ax} = \mathbf{c} \quad (4.1)$$

and

$$\mathbf{c}^l \leq \mathbf{c} \leq \mathbf{c}^u, \quad \mathbf{x}^l \leq \mathbf{x} \leq \mathbf{x}^u, \quad (4.2)$$

the dual optimality conditions

$$\mathbf{Hx} + \mathbf{g} = \mathbf{A}^T \mathbf{y} + \mathbf{z}, \quad \mathbf{y} = \mathbf{y}^l + \mathbf{y}^u \quad \text{and} \quad \mathbf{z} = \mathbf{z}^l + \mathbf{z}^u, \quad (4.3)$$

and

$$\mathbf{y}^l \geq 0, \quad \mathbf{y}^u \leq 0, \quad \mathbf{z}^l \geq 0 \quad \text{and} \quad \mathbf{z}^u \leq 0, \quad (4.4)$$

and the complementary slackness conditions

$$(\mathbf{Ax} - \mathbf{c}^l)^T \mathbf{y}^l = 0, \quad (\mathbf{Ax} - \mathbf{c}^u)^T \mathbf{y}^u = 0, \quad (\mathbf{x} - \mathbf{x}^l)^T \mathbf{z}^l = 0 \quad \text{and} \quad (\mathbf{x} - \mathbf{x}^u)^T \mathbf{z}^u = 0, \quad (4.5)$$

where the vectors  $\mathbf{y}$  and  $\mathbf{z}$  are known as the Lagrange multipliers for the general linear constraints, and the dual variables for the bounds, respectively, and where the vector inequalities hold component-wise. Primal-dual interior point methods iterate towards a point that satisfies these conditions by ultimately aiming to satisfy (4.1), (4.3) and

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(4.5), while ensuring that (4.2) and (4.4) are satisfied as strict inequalities at each stage. Appropriate norms of the amounts by which (4.1), (4.3) and (4.5) fail to be satisfied are known as the primal and dual infeasibility, and the violation of complementary slackness, respectively. The fact that (4.2) and (4.4) are satisfied as strict inequalities gives such methods their other title, namely interior-point methods.

The problem is solved in three phases. The goal of the first "initial feasible point" phase is to find a strictly interior point which is primal feasible, that is that (4.1) is satisfied. The GALAHAD package `GALAHAD_LSQP` is used for this purpose, and offers the options of either accepting the first strictly feasible point found, or preferably of aiming for the so-called "analytic center" of the feasible region. Having found such a suitable initial feasible point, the second "optimality" phase ensures that (4.1) remains satisfied while iterating to satisfy accurately (but not exactly) the dual feasibility (4.3) and complementary slackness (4.5) using the GALAHAD package `GALAHAD_QPB`. This optimality phase proceeds by approximately minimizing a sequence of barrier functions

$$q(\mathbf{x}) - \mu \left[ \sum_{i=1}^m \log(c_i - c_i^l) + \sum_{i=1}^m \log(c_i^u - c_i) + \sum_{j=1}^n \log(x_j - x_j^l) + \sum_{j=1}^n \log(x_j^u - x_j) \right],$$

for an appropriate sequence of positive barrier parameters  $\mu$  converging to zero while ensuring that (4.1) remain satisfied and that  $\mathbf{x}$  and  $\mathbf{c}$  are strictly interior points for (4.2). Note that terms in the above summations corresponding to infinite bounds are ignored, and that equality constraints are treated specially. See the documentation of `GALAHAD_QPB` for more details.

Since `GALAHAD_QPB` may not give a fully accurate solution, the last phase uses the GALAHAD package `GALAHAD_QPA` to refine this solution. This package uses a active/working-set method to solve the related  $\ell_1$  quadratic programming problem

$$\underset{\mathbf{x} \in \mathbb{R}^n}{\text{minimize}} \quad q(\mathbf{x}) + \rho_g v_g(\mathbf{x}) + \rho_b v_b(\mathbf{x}),$$

where the he infeasibilities

$$v_g(\mathbf{x}) = \sum_{i=1}^m \max(c_i^l - \mathbf{a}_i^T \mathbf{x}, 0) + \sum_{i=1}^m \max(\mathbf{a}_i^T \mathbf{x} - c_i^u, 0)$$

and

$$v_b(\mathbf{x}) = \sum_{j=1}^n \max(x_j^l - x_j, 0) + \sum_{j=1}^n \max(x_j - x_j^u, 0),$$

and  $\rho_g$  and  $\rho_b$  are appropriately chosen positive weights. See the documentation of `GALAHAD_QPA` for more details.

In order to make the solution as efficient as possible, the variables and constraints are reordered internally by the GALAHAD package `GALAHAD_QPP` prior to solution. In particular, fixed variables, and free (unbounded on both sides) constraints are temporarily removed.

If at any stage only equality constraints remain, a much simpler equality-constrained phase is performed, by GALAHAD package `GALAHAD_EQP`, in place of the three inequality-based phases described above.

## References:

The interior-point and working-set methods are described in detail in

A. R. Conn, N. I. M. Gould, D. Orban and Ph. L. Toint (1999). "A primal-dual trust-region algorithm for minimizing a non-convex function subject to general inequality and linear equality constraints". *Mathematical Programming* **87** 215–249.

N. I. M. Gould and Ph. L. Toint (2002). "An iterative working-set method for large-scale non-convex quadratic programming". *Applied Numerical Mathematics* **43** (1–2) 109–128.

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## 5 EXAMPLE OF USE

Suppose we wish to minimize  $\frac{1}{2}x_1^2 + x_2^2 + \frac{3}{2}x_3^2 + 4x_1x_3 + 2x_2 + 1$  subject to the the general linear constraints  $1 \leq 2x_1 + x_2 \leq 2$  and  $x_2 + x_3 = 2$ , and simple bounds  $-1 \leq x_1 \leq 1$  and  $x_3 \leq 2$ . Then, on writing the data for this problem as

$$\mathbf{H} = \begin{pmatrix} 1 & & & \\ & 2 & & \\ & & 3 & \\ 4 & & & \end{pmatrix}, \quad \mathbf{g} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{x}^l = \begin{pmatrix} -1 \\ -\infty \\ -\infty \end{pmatrix} \quad \text{and} \quad \mathbf{x}^u = \begin{pmatrix} 1 \\ \infty \\ 2 \end{pmatrix},$$

and

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & & \\ & 1 & 1 & \end{pmatrix}, \quad \mathbf{c}^l = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \text{and} \quad \mathbf{c}^u = \begin{pmatrix} 2 \\ \end{pmatrix}$$

in sparse co-ordinate format, we may use the following code:

```
! THIS VERSION: GALAHAD 2.2 - 23/04/2008 AT 16:30 GMT.
PROGRAM GALAHAD_QPC_EXAMPLE
USE GALAHAD_QPC_double ! double precision version
IMPLICIT NONE
INTEGER, PARAMETER :: wp = KIND(1.0D+0) ! set precision
REAL (KIND = wp), PARAMETER :: infinity = 10.0_wp ** 20
TYPE (QPT_problem_type) :: p
TYPE (QPC_data_type) :: data
TYPE (QPC_control_type) :: control
TYPE (QPC_inform_type) :: inform
INTEGER :: s
INTEGER, PARAMETER :: n = 3, m = 2, h_ne = 4, a_ne = 4
INTEGER, ALLOCATABLE, DIMENSION(:) :: C_stat, B_stat
! start problem data
ALLOCATE(p%G(n), p%X_l(n), p%X_u(n))
ALLOCATE(p%C(m), p%C_l(m), p%C_u(m))
ALLOCATE(p%X(n), p%Y(m), p%Z(n))
ALLOCATE(B_stat(n), C_stat(m))
p%new_problem_structure = .TRUE. ! new structure
p%n = n ; p%m = m ; p%f = 1.0_wp ! dimensions & objective constant
p%G = (/ 0.0_wp, 2.0_wp, 0.0_wp /) ! objective gradient
p%C_l = (/ 1.0_wp, 2.0_wp /) ! constraint lower bound
p%C_u = (/ 2.0_wp, 2.0_wp /) ! constraint upper bound
p%X_l = (/ -1.0_wp, -infinity, -infinity /) ! variable lower bound
p%X_u = (/ 1.0_wp, infinity, 2.0_wp /) ! variable upper bound
p%X = 0.0_wp ; p%Y = 0.0_wp ; p%Z = 0.0_wp ! start from zero
! sparse co-ordinate storage format
CALL SMT_put(p%H%type, 'COORDINATE', s) ! Specify co-ordinate
CALL SMT_put(p%A%type, 'COORDINATE', s) ! storage for H and A
ALLOCATE(p%H%val(h_ne), p%H%row(h_ne), p%H%col(h_ne))
ALLOCATE(p%A%val(a_ne), p%A%row(a_ne), p%A%col(a_ne))
p%H%val = (/ 1.0_wp, 2.0_wp, 3.0_wp, 4.0_wp /) ! Hessian H
p%H%row = (/ 1, 2, 3, 3 /) ! NB lower triangle
p%H%col = (/ 1, 2, 3, 1 /) ; p%H%ne = h_ne
p%A%val = (/ 2.0_wp, 1.0_wp, 1.0_wp, 1.0_wp /) ! Jacobian A
p%A%row = (/ 1, 1, 2, 2 /)
p%A%col = (/ 1, 2, 2, 3 /) ; p%A%ne = a_ne
! problem data complete
CALL QPC_initialize(data, control, inform) ! Initialize control parameters
control%infinity = infinity ! Set infinity
! control%print_level = 1
```

---

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```

! control%QPA_control%print_level = 1
! control%QPB_control%print_level = 1
CALL QPC_solve(p, C_stat, B_stat, data, control, inform) ! Solve problem
IF (inform%status == 0) THEN
 WRITE(6, "(' QPC: ', I0, ' QPA and ', I0, ' QPB iterations ', /,
 & ' Optimal objective value = ',
 & ES12.4, /, ' Optimal solution = ', (5ES12.4))")
 inform%QPA_inform%iter, inform%QPB_inform%iter, inform%obj, p%X
ELSE
 WRITE(6, "(' QPC_solve exit status = ', I6) ") inform%status
END IF
CALL QPC_terminate(data, control, inform) ! delete internal workspace
END PROGRAM GALAHAD_QPC_EXAMPLE

```

This produces the following output:

```

QPC: -1 QPA and 7 QPB iterations
Optimal objective value = 5.4459E+00
Optimal solution = -5.4054E-02 1.1081E+00 8.9189E-01

```

The same problem may be solved holding the data in a sparse row-wise storage format by replacing the lines

```

! sparse co-ordinate storage format
...
! problem data complete

```

by

```

! sparse row-wise storage format
CALL SMT_put(p%H%type, 'SPARSE_BY_ROWS', s) ! Specify sparse-by-row
CALL SMT_put(p%A%type, 'SPARSE_BY_ROWS', s) ! storage for H and A
ALLOCATE(p%H%val(h_ne), p%H%col(h_ne), p%H%ptr(n + 1))
ALLOCATE(p%A%val(a_ne), p%A%col(a_ne), p%A%ptr(m + 1))
p%H%val = (/ 1.0_wp, 2.0_wp, 3.0_wp, 4.0_wp /) ! Hessian H
p%H%col = (/ 1, 2, 3, 1 /) ! NB lower triangular
p%H%ptr = (/ 1, 2, 3, 5 /) ! Set row pointers
p%A%val = (/ 2.0_wp, 1.0_wp, 1.0_wp, 1.0_wp /) ! Jacobian A
p%A%col = (/ 1, 2, 2, 3 /)
p%A%ptr = (/ 1, 3, 5 /) ! Set row pointers
! problem data complete

```

or using a dense storage format with the replacement lines

```

! dense storage format
CALL SMT_put(p%H%type, 'DENSE', s) ! Specify dense
CALL SMT_put(p%A%type, 'DENSE', s) ! storage for H and A
ALLOCATE(p%H%val(n * (n + 1) / 2))
ALLOCATE(p%A%val(n * m))
p%H%val = (/ 1.0_wp, 0.0_wp, 2.0_wp, 4.0_wp, 0.0_wp, 3.0_wp /) ! Hessian
p%A%val = (/ 2.0_wp, 1.0_wp, 0.0_wp, 0.0_wp, 1.0_wp, 1.0_wp /) ! Jacobian
! problem data complete

```

respectively.

If instead **H** had been the diagonal matrix

$$\mathbf{H} = \begin{pmatrix} 1 & & \\ & 0 & \\ & & 3 \end{pmatrix}$$

---

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but the other data is as before, the diagonal storage scheme might be used for **H**, and in this case we would instead

```
CALL SMT_put(prob%H%type, 'DIAGONAL', s) ! Specify dense storage for H
ALLOCATE(p%H%val(n))
p%H%val = (/ 1.0_wp, 0.0_wp, 3.0_wp /) ! Hessian values
```

Notice here that zero diagonal entries are stored.